ECE 4800 – PROJECT 1:

ANALYSIS OF A DISCRETE-ELEMENT TRANSMISSION LINE MODEL

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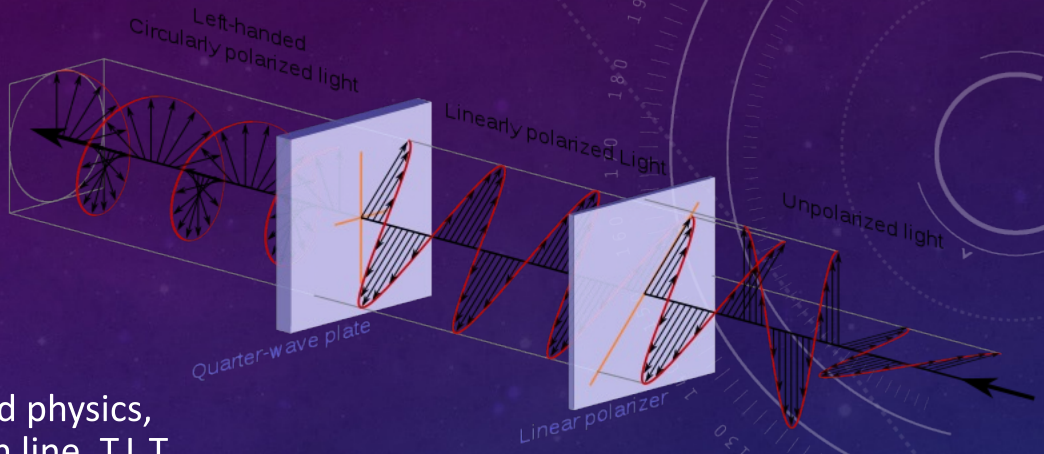
INTRODUCTION:

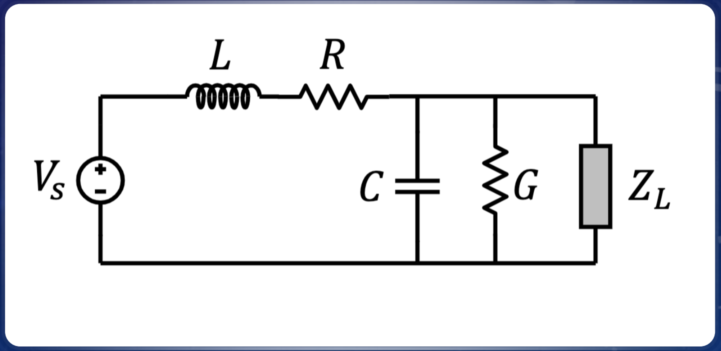


Transmission line theory is a fundamental aspect of electrical engineering and physics, which focuses on how electromagnetic waves propagate along a transmission line. T.L.T. is important in understanding and designing systems for transmitting electrical energy and signals over long distances, such as in power distribution networks and telecommunications.

Throughout ECE 4800, we have studied in detail how electromagnetic waves propagate and reflect through transmission lines.

After initial study of idealized (lossless) transmission lines, we focused on understanding the implications of wave propagation through non-deal, practical (lossy) transmission lines. Through study of Heaviside's Telegrapher's Equations, we've learned about the consequences of propagating signals through practical transmission lines, and the important considerations we must take when measuring and analyzing a wave's behavior under these conditions.

Circuit model of a practical transmission line:



PROJECT OBJECTIVES:

The goal of this project was to further our theoretical understanding of transmission lines and their properties by realizing a discrete-element transmission line model and experimentally measuring many of the circuit's properties.

Measurements conducted:

1. Experimental measurement of resistance, capacitance and inductance for each of discrete element used.
2. Measuring wavelength, standing wave ration, reflection coefficient, phase of reflection, and load impedance for each of the following four cases:
    I. Transmission Line with matched load
    II. Transmission Line with short-circuit at load
    III. Transmission Line with open circuit at load
    IV. Transmission Line with complex impedance load
3. Transmission Line's response to a rectangular pulse
4. Frequency Response of Transmission line

MEASURING CIRCUIT ELEMENTS USED: RESISTORS

Our first task was to measure the experimental value of capacitance, inductance, and resistance of all the circuit elements used in the fulfillment of this experiment.

Throughout the experiment, the source impedance, ZG, was maintained at approximately 50Ω.

Whenever a 50Ω load was used, the measured ZL (below) was used.

Component:	Theoretical Value:	Experimental Value:	Percent Error:
Z_G	50	49.063	-1.874%
Z_L	50	49.780	-0.44%

MEASURING CIRCUIT ELEMENTS USED: INDUCTORS AND CAPACITORS

For our project, we employed a straightforward yet effective method to determine the inductance and capacitance values of our circuit components. This was achieved through the utilization of the circuit depicted in Figure 5.

This method involved measuring the voltage drop across a 50Ω resistor when applying a sinusoidal signal of 1V amplitude at frequencies between 10kHz-80kHz (10kHz step). This would then allow us to experimentally calculate the current flowing through the simple RL/RC circuit.

We then measured the voltage drop across the capacitor/inductor in question at the various input frequencies and tabulated the results.

This process was repeated for the 15 inductors and 15 capacitors that were used to build our final model, and averaged the voltage drop values at each frequency. These results are shown in the following two tables.

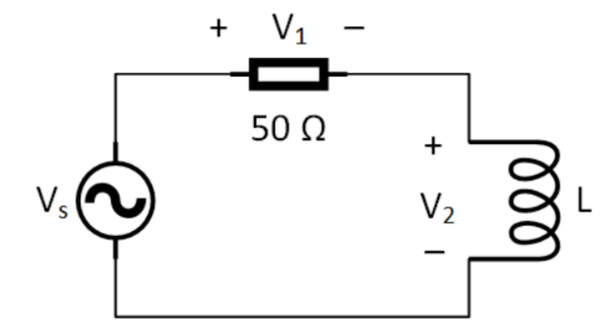


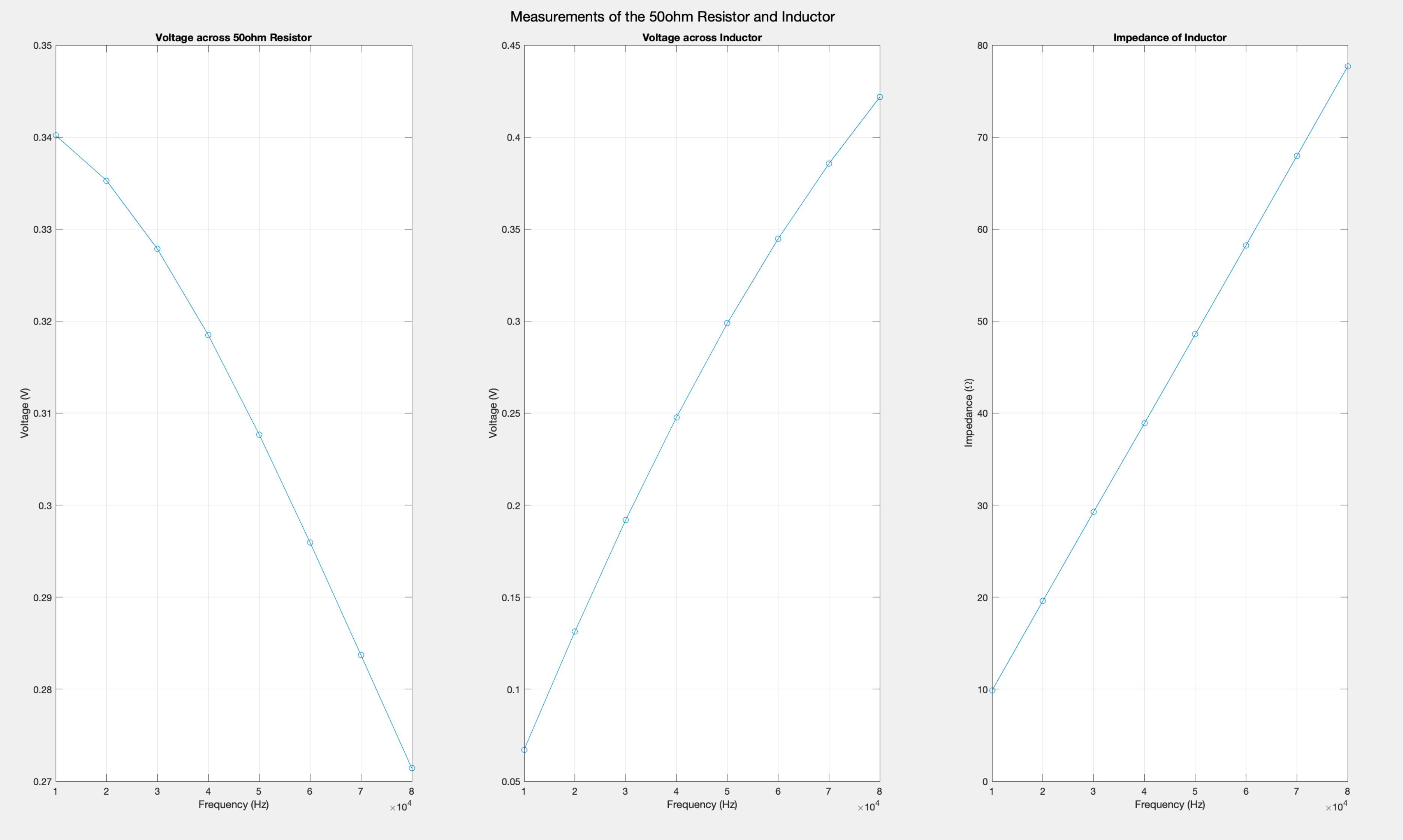
Fig. 5: Measurement of an inductance.

MEASURING CIRCUIT ELEMENTS USED: INDUCTORS AND CAPACITORS

Input Frequency of 1V Sine Input:	Average Voltage Drop Across 50Ω Resistor:	Average Voltage Drop Across Inductors:
10kHz	0.34018V	0.06716V
20kHz	0.33527V	0.13136V
30kHz	0.32783V	0.192015V
40kHz	0.31848V	0.24778V
50kHz	0.30768V	0.29885V
60kHz	0.29595V	0.34472V
70kHz	0.28372V	0.38555V
80kHz	0.27141V	0.42164V

Input Frequency of 1V Sine Input:	Average Voltage Drop Across 50Ω Resistor:	Average Voltage Drop Across Capacitors:
10kHz	0.13135V	0.65799V
20kHz	0.22237V	0.55669V
30kHz	0.27479V	0.45913V
40kHz	0.30442V	0.38319V
50kHz	0.32192V	0.32248V
60kHz	0.33302V	0.27777V
70kHz	0.34035V	0.24305V
80kHz	0.34577V	0.21559V

PLOTTED VALUES FOR RL CIRCUIT:

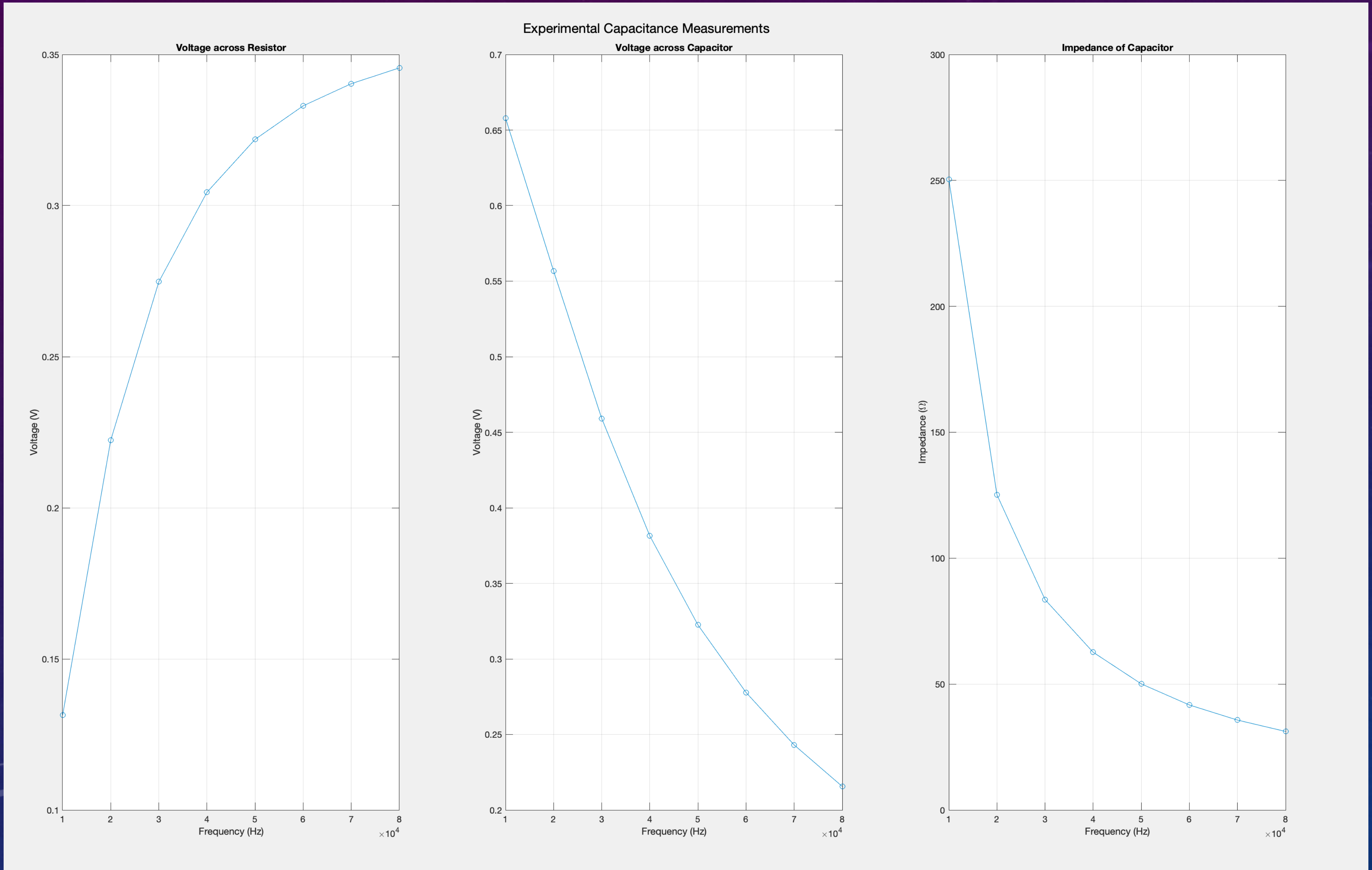


These three charts show the measured values for: average voltage drop across the 50Ω resistor, average voltage drop across the inductor, and the average calculated impedance of the inductors, as a function of input frequency.

The values in the right-most chart were calculated using the following formula:

$$ZL = jwL$$

PLOTTED VALUES FOR RC CIRCUIT:



These three charts show the measured values for: average voltage drop across the 50Ω resistor, average voltage drop across the capacitor, and the average calculated impedance of the capacitor, as a function of input frequency.

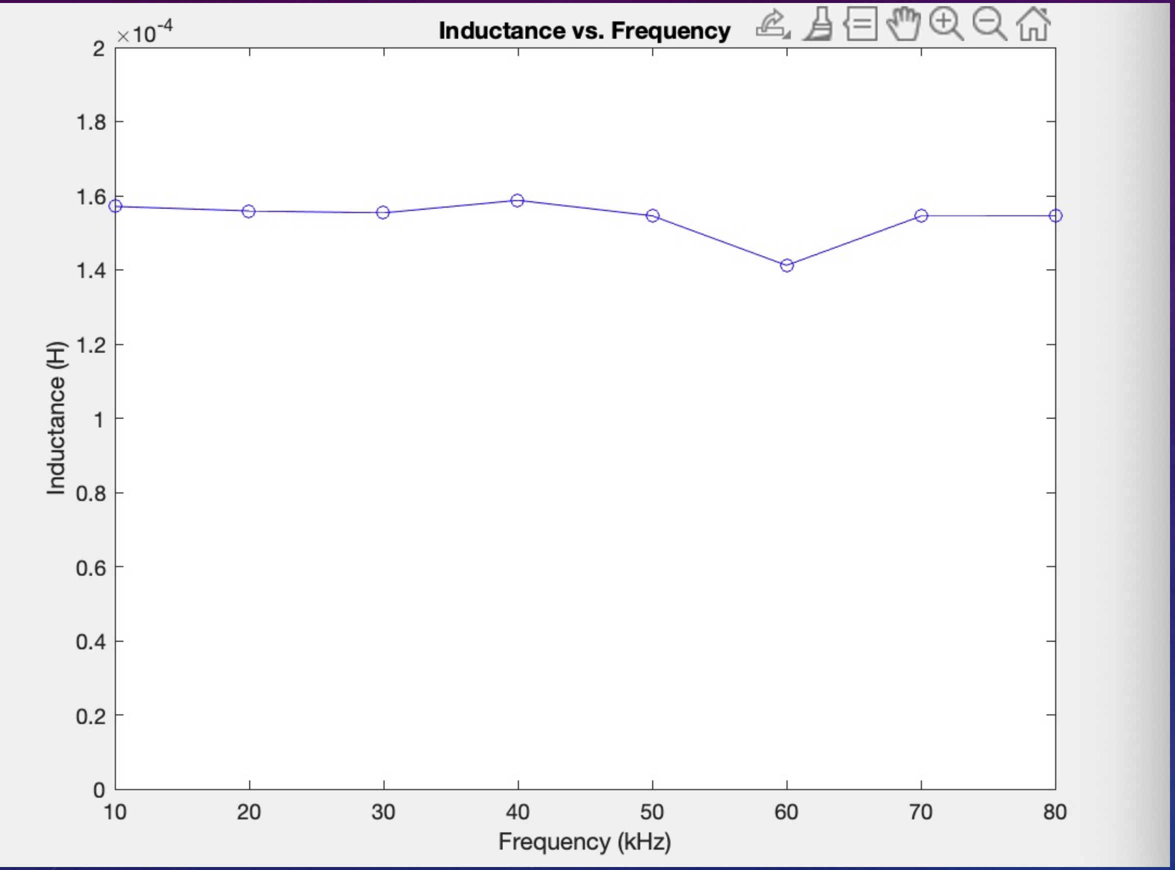
The values in the right-most chart were calculated using the following formula:

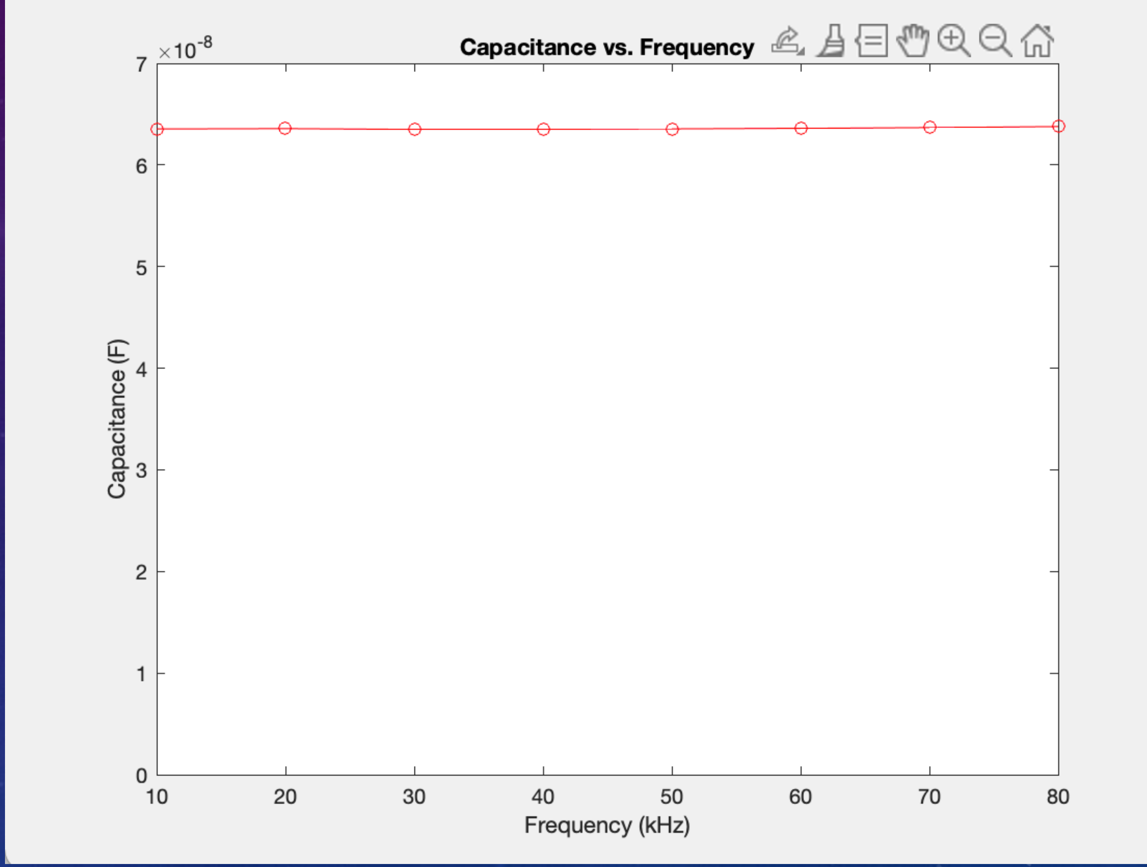
$$Zc = 1/(jwC)$$

USING MEASURED IMPEDANCES TO CALCULATE FOR INDUCTANCE, CAPACITANCE

- We know that impedance is a complex number, with a real and imaginary component. However, without access to tools such as a network analyzer, it was difficult to determine how much of the impedance measured across the inductors and capacitors was due to resistance vs. reactance.
- The following calculations for average inductance and capacitance were calculated with the assumption that the measured average impedances were completely imaginary.
- We know this not to be the case, as all real-world components carry some amount of parasitic loss. However, careful attention was paid when choosing which inductors and capacitors to purchase, in attempts to minimize parasitic resistive loss as much as possible.
- Links to the capacitors, inductors purchased:
  - Capacitors: https://www.mouser.com/ProductDetail/667-ECW-H10623JVB
  - Inductors: https://www.mouser.com/ProductDetail/871-B82144F2154J000

USING MEASURED IMPEDANCES TO CALCULATE FOR INDUCTANCE, CAPACITANCE





USING MEASURED IMPEDANCES TO CALCULATE FOR INDUCTANCE, CAPACITANCE

After measuring the inductance and capacitance at each frequency, the average inductance and capacitance was calculated:

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The average inductance L is: 153.993618 microHenries
The average capacitance C is: 0.063593 microFarrads
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Parameter:	Theoretical Value:	Experimental Value:	Percent Error:
Inductance	150 uH	153.994 uH	+2.66%
Capacitance	.062 uF	.0636 uF	+2.58%

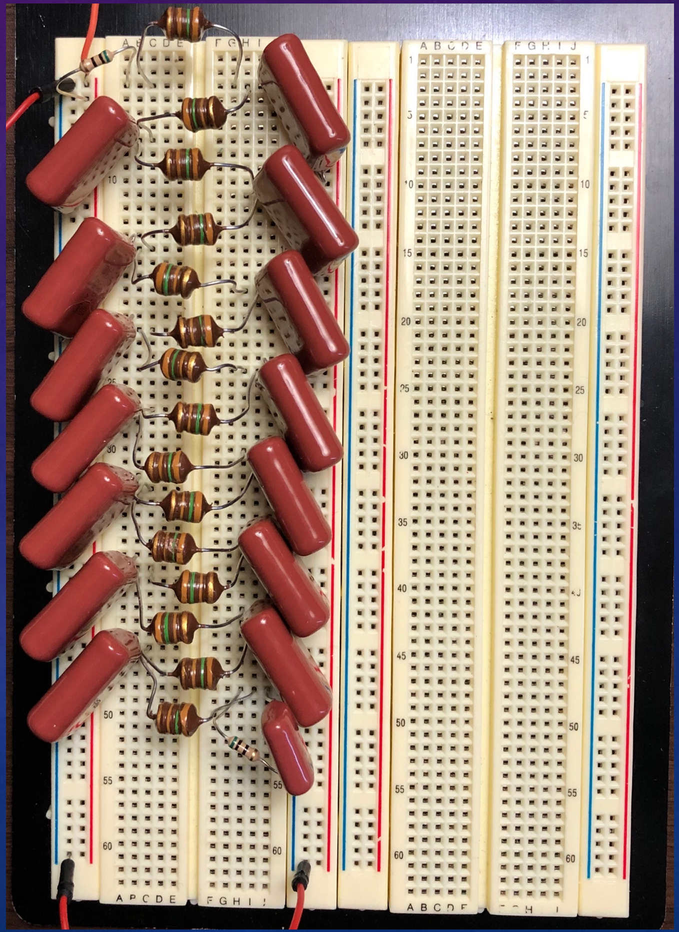
As can be observed from the table above, the inductors and capacitors were measured to function very accurately, with less than a 5% percent of error.

BUILDING THE CIRCUIT:

- Once I was certain the components were accurate and trustworthy, I implemented the discrete-element model.

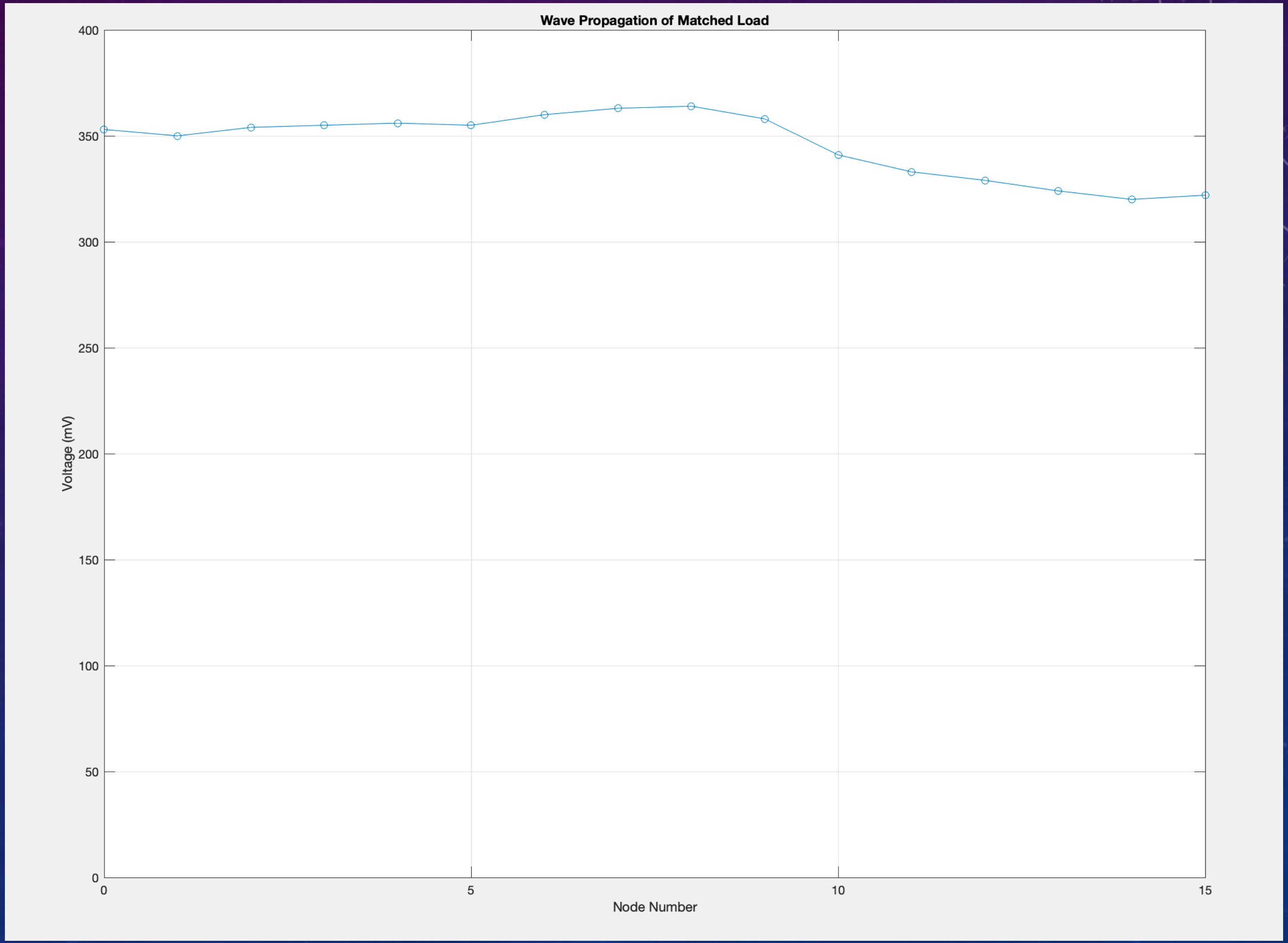
- Careful attention was paid to making sure that each node was at an equal distance from one another.

- In total, the artificial transmission line was made up of 15 capacitors, 15 inductors, 1 50-ohm resistor, and a varying resistance at the load.



MATCHED LOAD - RESULTS:

50 ohm load	
node	voltage ml
0	353
1	350
2	354
3	355
4	356
5	355
6	360
7	363
8	364
9	358
10	341
11	333
12	329
13	324
14	320
15	322



Matched Load Calculations and Results:

- In a transmission line with a matched load, where the load impedance equals the characteristic impedance of the line, there should ideally be no reflected waves. This is because all the energy from the incident wave is absorbed by the load, and there is no mismatch to cause any reflection.
- In practice, a source and load impedance will never perfectly match, and the wave may decay as it encounters parasitic loss across the line.

- With an ideal matched load, there are no standing waves, as all the power is transmitted to the load. So, the SWR should be 1, which corresponds to a flat line with no variation in amplitude along the line.

- With an ideal matched load, there are no standing waves so the SWR should be 1, which corresponds to a flat line with no variation in amplitude along the line.

MATCHED LOAD CALCULATIONS AND RESULTS:

Matched Load Results:

$$C := 0.063593 \cdot \mu F \qquad L := 153.993618 \cdot \mu H$$

$$f := 15 \cdot 10^3 \cdot \frac{1}{s}$$

$$Vmax := 364 \cdot mV \qquad Vmin := 320 \cdot mV \qquad Z_0 := \left(\frac{L}{C}\right)^{\frac{1}{2}} = 49.209\ \Omega$$

1.

$$SWR_1 := \frac{|Vmax|}{|Vmin|} = 1.1375$$

$$\Gamma_1 := \frac{(SWR_1 - 1)}{(SWR_1 + 1)} = 0.064327$$

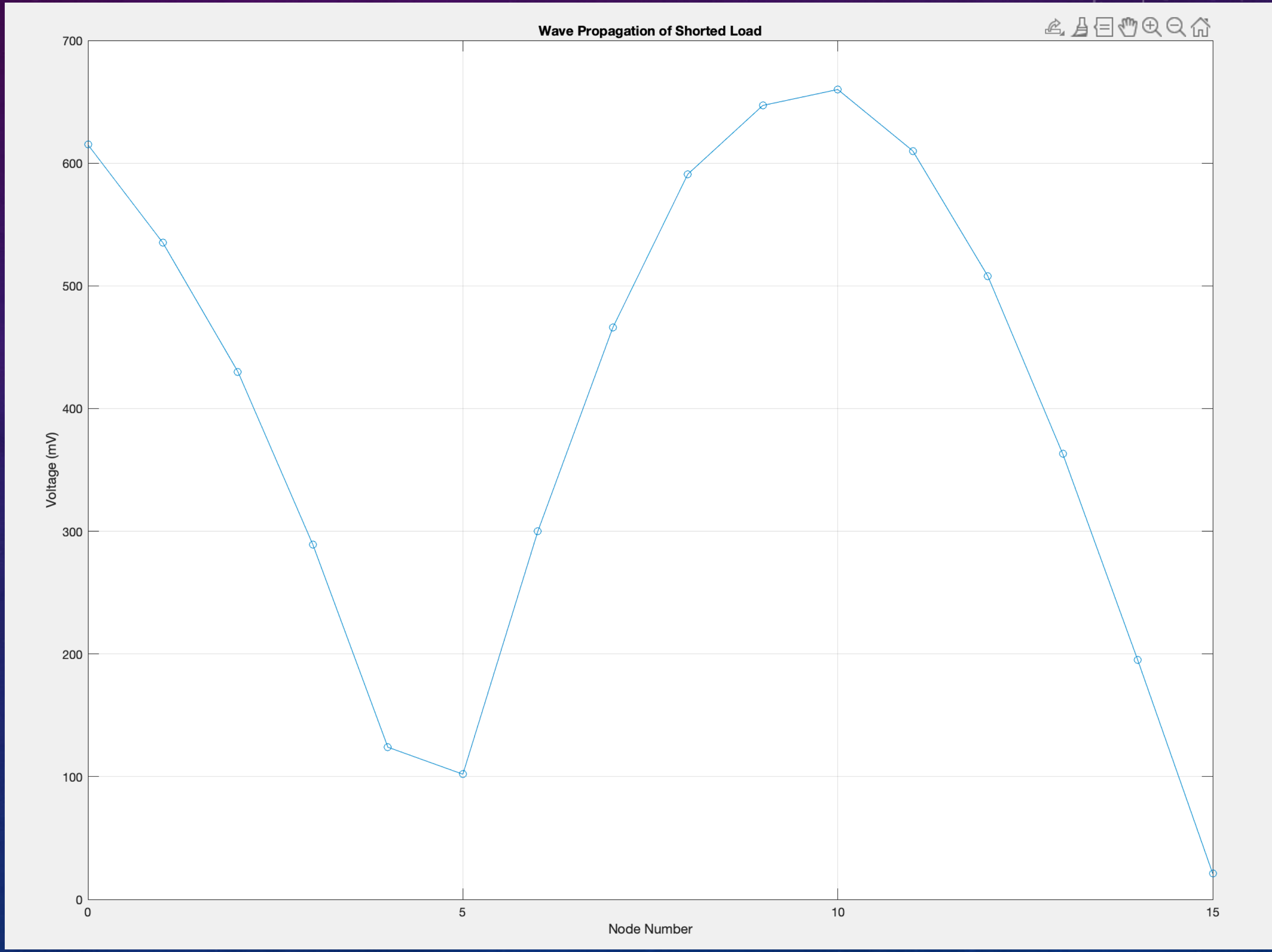
$$Z_{L1} := Z_0 \cdot \frac{(1 + \Gamma_1)}{(1 - \Gamma_1)} = 55.976\ \Omega$$

As can be seen in the figure to the left, the experimental values of SWR, reflection coefficient, and load impedance are fairly close to the theoretical expectations.

Property	Theoretical Value	Experimental Value	Percent Error:
Wavelength	N/A	N/A	N/A
SWR	1	1.1375	13.75%
Reflection Coefficient	0	0.064327	inf
Phase of Reflected Wave	N/A	N/A	N/A
Load Impedance	50Ω	55.976Ω	11.53%

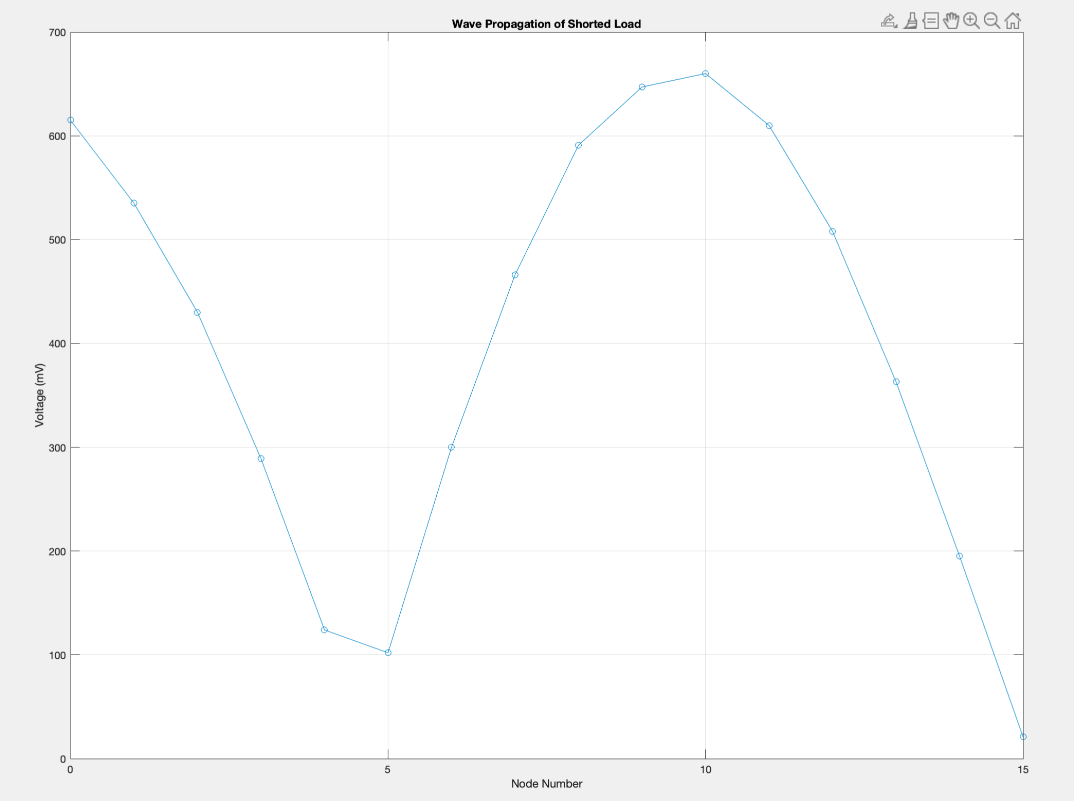
SHORT-CIRCUIT AT LOAD - RESULTS

node	short at load(mV)
0	615
1	535
2	430
3	289
4	124
5	102
6	300
7	466
8	591
9	647
10	660
11	610
12	508
13	363
14	195
15	21



SHORT-CIRCUIT AT LOAD - RESULTS

- With a short circuit load, the SWR should theoretically infinite because Vmax/Vmin is divided by 0.
- The absolute value of the reflection coefficient should be 1 as all the incident wave is reflected back.
- The phase of the reflection coefficient should be $-\pi$ radians, as a short circuit introduces a half-wavelength phase shift (delay) in the reflected wave.
- The load impedance is 0 ohms due to the short circuit.



SHORT-CIRCUIT AT LOAD - RESULTS

Calculating The theoretical wavelength:

$$v_{prop.} := \frac{1}{(150\ \mu H \cdot .062 \cdot \mu F)^{\frac{1}{2}}} = (3.279 \cdot 10^{5})\ \frac{1}{s}$$

$$f := 15 \cdot 10^{3} \cdot \frac{1}{s}$$

$$\lambda_{theoretical.} := \frac{v_{prop.}}{f} = 21.861$$

Shorted Load Results:

$$Vmax := 660 \cdot mV \qquad Vmin := 21 \cdot mV$$

$$Z_0 := \left(\frac{L}{C}\right)^{\frac{1}{2}} = 49.209\ \Omega$$

1.

$$SWR_1 := \frac{|Vmax|}{|Vmin|} = 31.4285714$$

$$\lambda := 2 \cdot (15 - 5) = 20$$

$$\Gamma_1 := \frac{(SWR_1 - 1)}{(SWR_1 + 1)} = 0.938326$$

$$\theta_r := -4 \cdot \pi \cdot \frac{(15 - 10)}{\lambda} = -3.142$$

$$Z_{L1} := Z_0 \cdot \frac{(1 + \Gamma_1)}{(1 - \Gamma_1)} = (1.547 \cdot 10^{3})\ \Omega$$

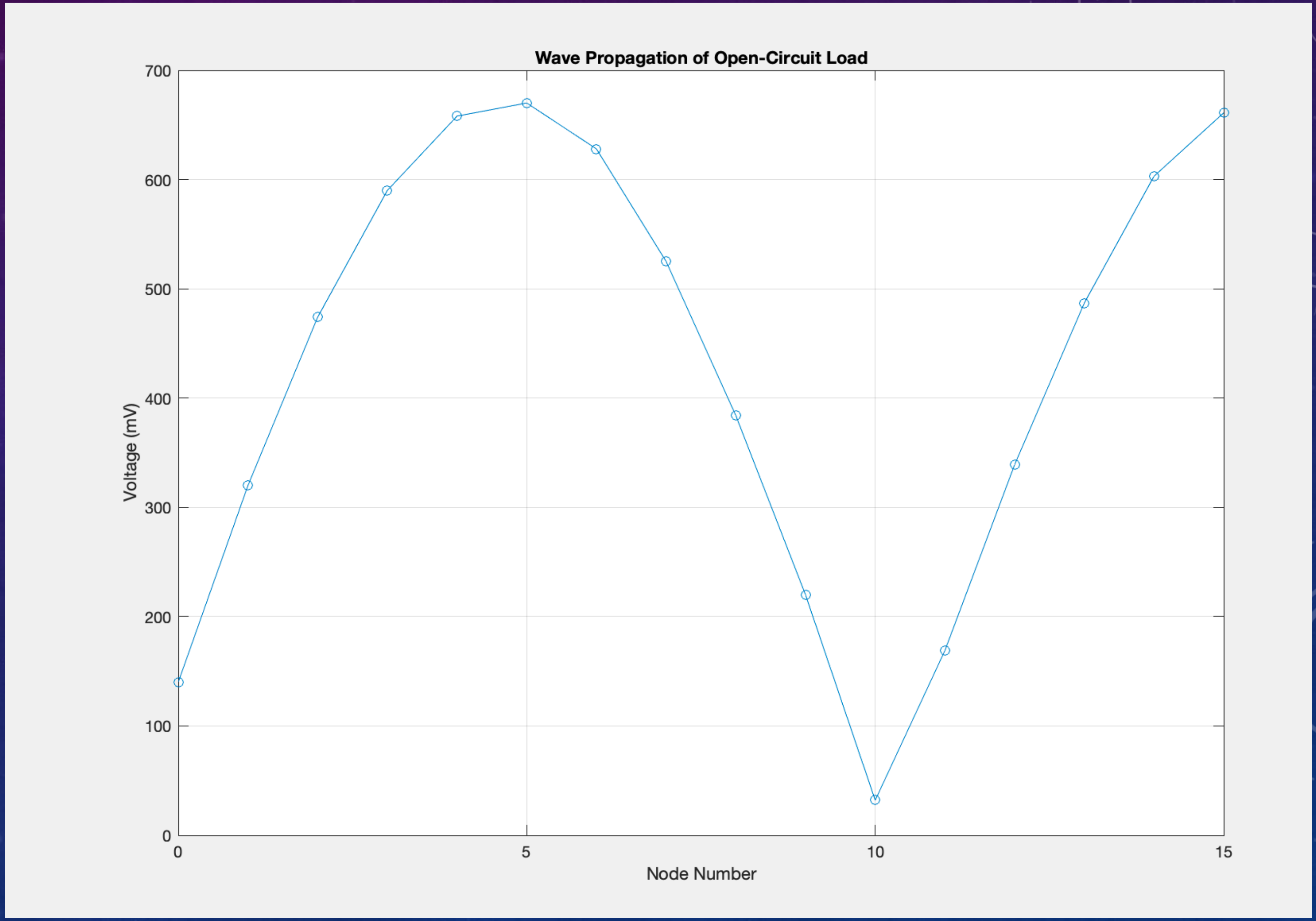
Note: The theoretical wavelength measured in the results above is used to compare all other wavelengths going forward.

As can be seen in the figure to the left, our experimental values of wavelength, reflection coefficient, and phase of reflected wave are accurately approximated to the theoretical expectations. The SWR is, in theory, ‘close’, as it approximates infinity. However, our load impedance does not approximate to 0, because our reflection coefficient is not exactly 1.

Property	Theoretical Value	Experimental Value	Percent Error:
Wavelength	20	21.861	-8.51%
SWR	inf	31.428	inf
Reflection Coefficient	1	0.938	-6.2%
Phase of Reflected Wave	-3.14159	-3.142	0.013%
Load Impedance	0	1,547Ω	inf

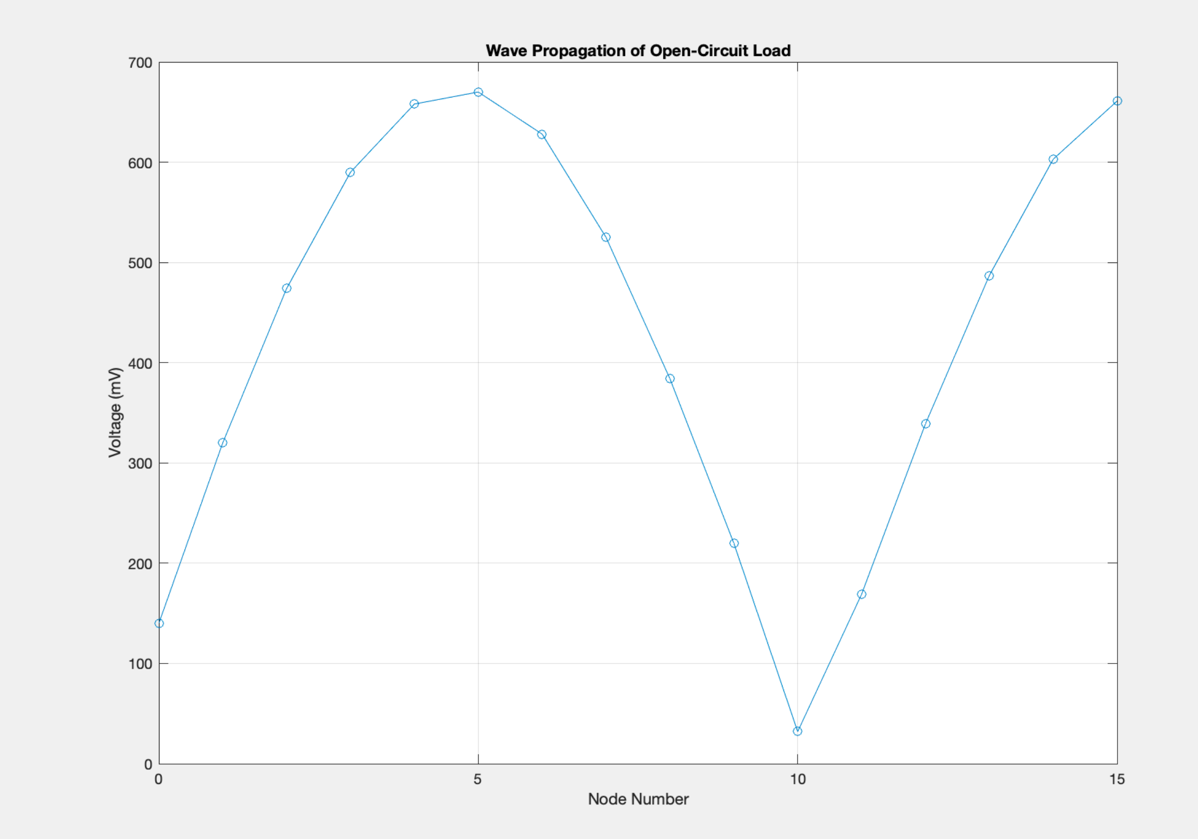
OPEN CIRCUIT AT LOAD RESULTS:

node	open circuit (mV)
0	140
1	320
2	474
3	590
4	658
5	670
6	628
7	525
8	384
9	220
10	32
11	169
12	339
13	487
14	603
15	661



OPEN CIRCUIT AT LOAD RESULTS:

- With an open-circuit load, the SWR should also be theoretically infinite because Vmax/Vmin is divided by 0.
- The reflection coefficient should be 1.
- The phase of the reflection coefficient should be 0, or integer multiples of 2π
- The load impedance is theoretically infinite, as an open circuit has ‘infinite’ resistance.



OPEN CIRCUIT AT LOAD - RESULTS

Open Load Results: $\lambda := 2 \cdot (15 - 5) = 20$ *nodes*

$Vmax := 670 \cdot mV$ $Vmin := 32 \cdot mV$

$$SWR_3 := \frac{|Vmax|}{|Vmin|} = 20.9375$$

$$\Gamma_3 := \frac{(SWR_3 - 1)}{(SWR_3 + 1)} = 0.908832$$

$$\theta_r := -4 \cdot \pi \cdot \frac{15 - 5}{\lambda} = -6.283$$

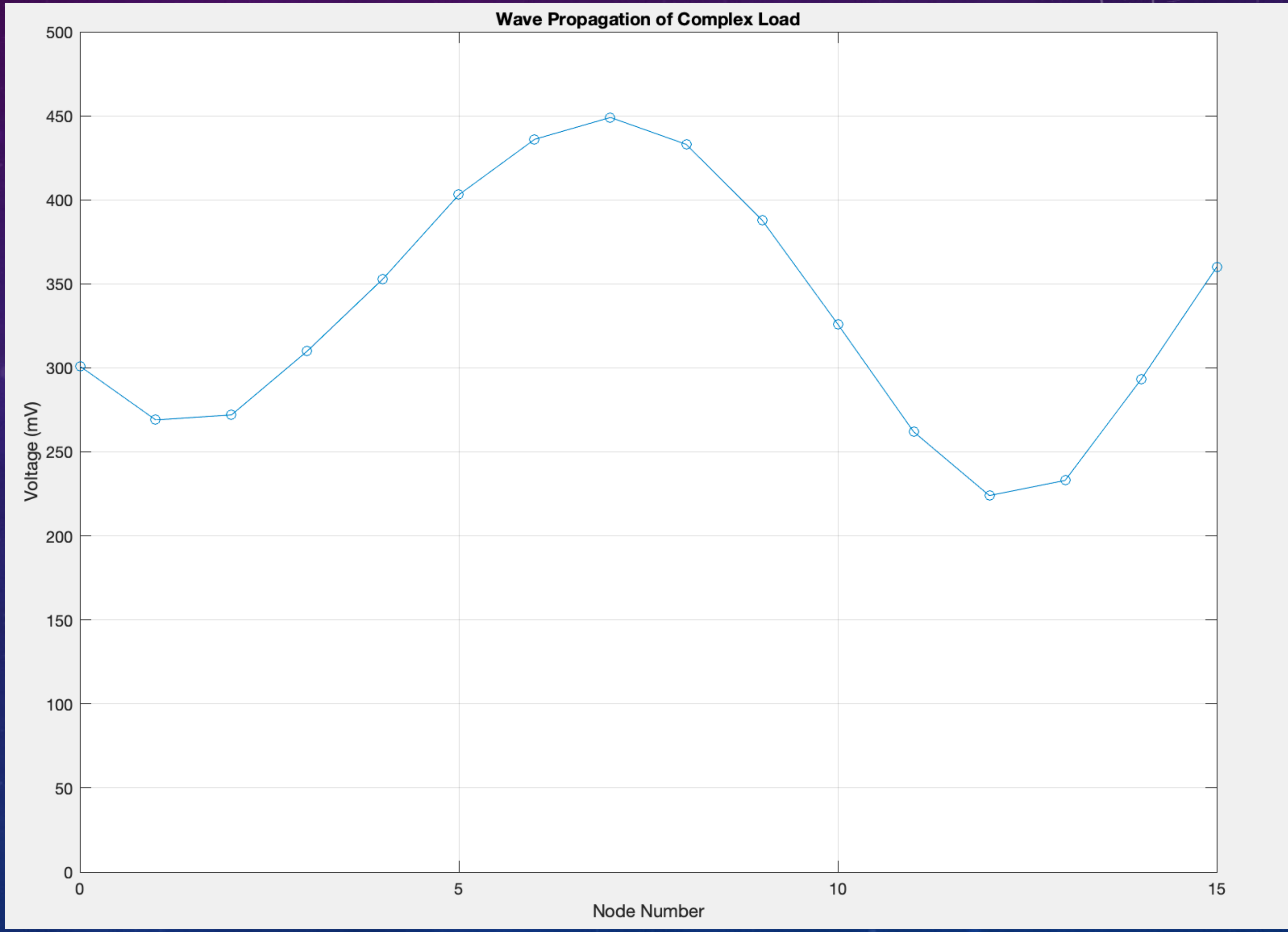
$$Z_{L3} := Z_0 \cdot \frac{(1 + \Gamma_3)}{(1 - \Gamma_3)} = 1030.319\ \Omega$$

- The experimental values of wavelength, reflection coefficient, and phase of reflected wave are accurately approximated to the theoretical expectations.
- The SWR is, in theory, ‘close’, as it approximates infinity.
- The reflection coefficient approximates 1 closely.
- The phase difference is nearly identical to theoretical assumptions.
- the load impedance is also very large, as it ‘approaches infinity’ but is affected by the non-ideal values of SWR and reflection coefficient.

Property	Theoretical Value	Experimental Value	Percent Error:
Wavelength	20	21.861	-8.51%
SWR	inf	20.9375	inf
Reflection Coefficient	1	0.909	-9.1%
Phase of Reflected Wave	0,2π	-6.283	-0.003%
Load Impedance	inf	1,030.319Ω	inf

COMPLEX LOAD RESULTS

node	
0	complex load(mV)
1	301
2	269
3	272
4	310
5	353
6	403
7	436
8	449
9	433
10	388
11	326
12	262
13	224
14	233
15	293
	360



Complex Load Calculations:

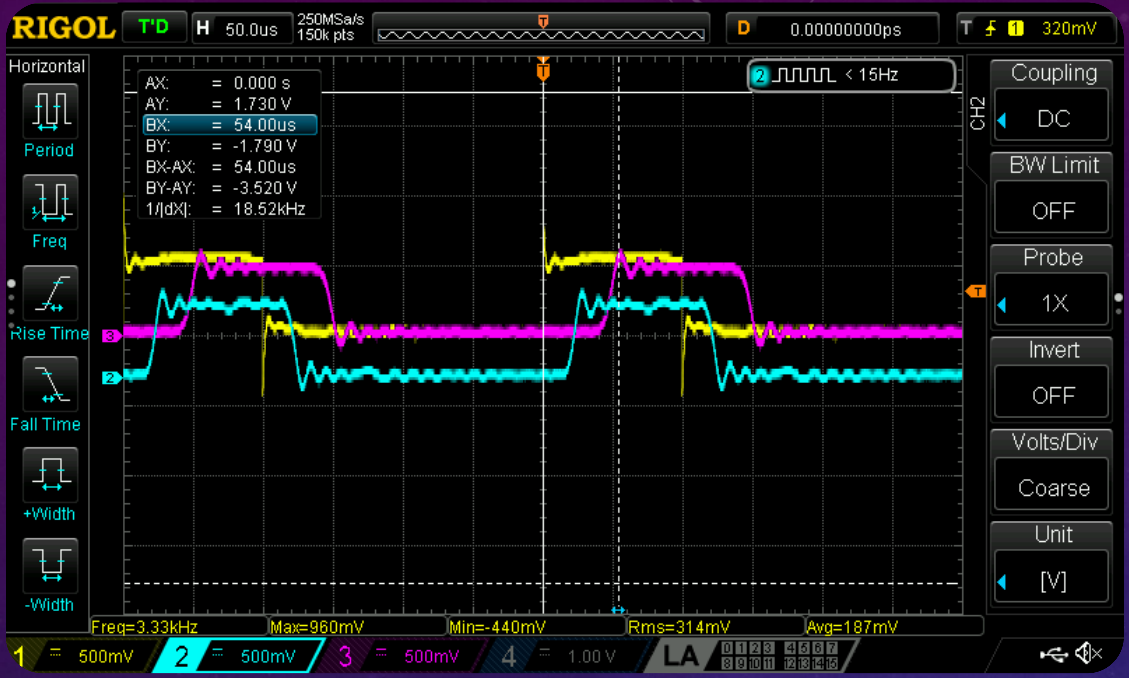
Complex Load Calculations

$$Zmax := 15 - 7 \quad Vmax := 449 \quad Vmin := 224 \quad L := 153.993618 \quad C := .063593$$

$$\lambda := 2\cdot(12 - 1) \rightarrow 22 \qquad SWR := \frac{Vmax}{Vmin} \text{ float}, 4 \rightarrow 2.004$$

$$\Gamma := \frac{SWR - 1}{SWR + 1} \rightarrow .33422103861517976032 \qquad \theta r := \frac{-4\cdot\pi\cdot Zmax}{\lambda} \text{ float}, 5 \rightarrow -4.5695$$

$$Zo := \sqrt{\frac{L}{C}} \rightarrow 49.209245316975110566 \qquad ZL := Zo\cdot\frac{1 + \Gamma}{1 - \Gamma} \rightarrow 98.615327615218173655$$



$$T := 0.3 \cdot ms$$

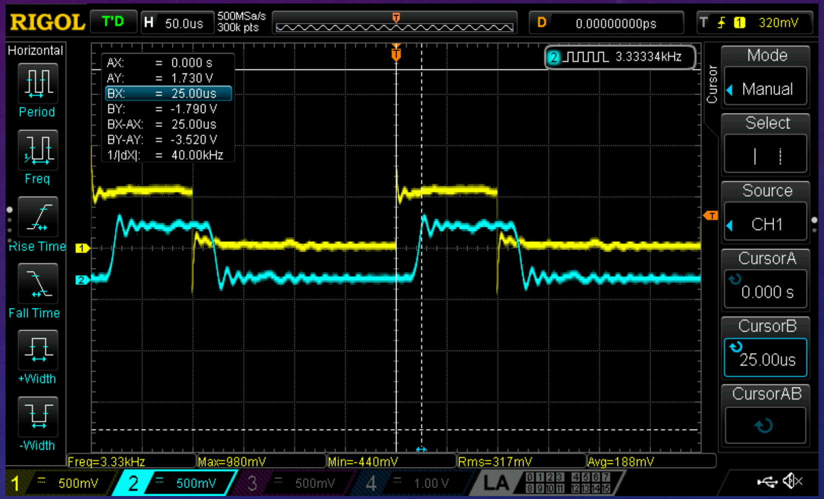
$$f := \frac{1}{T} = \left(3.333333 \cdot 10^{3}\right) \frac{1}{s}$$

$$DutyCycle := \frac{0.1 \cdot ms}{T} = 0.333 \quad \%$$

RESPONSE TO RECTANGULAR IMPULSE:

- The next part of the analysis was to determine the delay of a 1V amplitude square pulse on 2 separate points of the transmission line; nodes 7 and 15, in reference to the input voltage signal.
- The oscilloscope capture provided shows the voltage signal at these three different points on the transmission line. The equations provided show the period, frequency, and duty cycle of the input signal.

RESPONSE TO RECTANGULAR IMPULSE, NODE 7



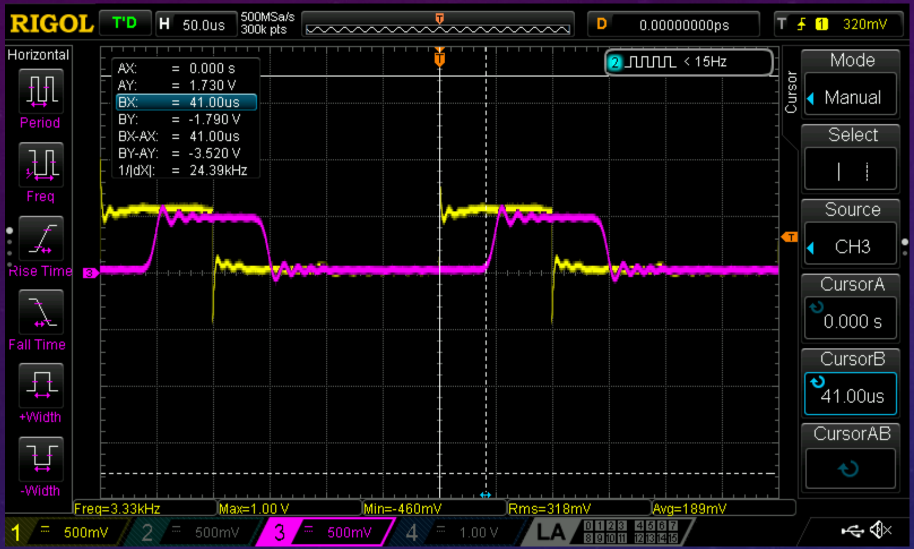
$$\tau_{1Theoretical} := \frac{7}{v_{prop.}} = \left(2.191 \cdot 10^{-5}\right)\ s$$

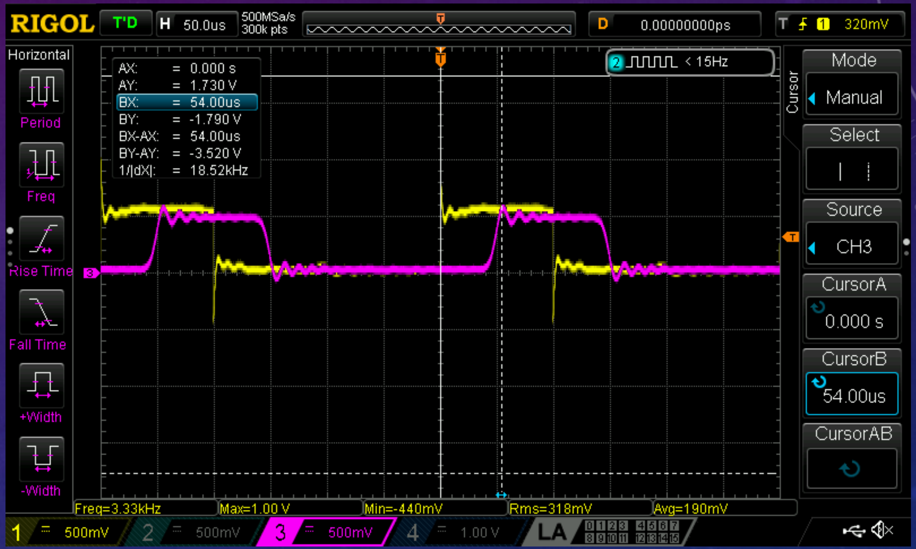
$$\tau_1 := \frac{16\ \mu s + 25\ \mu s}{2} = \left(2.05 \cdot 10^{-5}\right)\ s \qquad \tau_1 = 20.5 \cdot \mu s$$

RESPONSE TO RECTANGULAR IMPULSE, NODE 7

- The previous slide shows 2 separate oscilloscope captures of the input signal and the voltage signal across node 7.
- The time-delay between the two signals was averaged between the two captures, which led to the experimental node 7 value for Tau.
- The theoretical value of Tau was calculated using the ideal values of inductance and capacitance, calculating the wave velocity as :$(1/\sqrt{C * L})$, and then dividing the distance of the node (7) to find theoretical Tau.
- As can be seen in the previous slide, the experimental and theoretical Tau were very close, showing once again an accurate implementation of the transmission line model.
- The percent error between theoretical and experimental Tau was -6.435%

RESPONSE TO RECTANGULAR IMPULSE, NODE 15





$$\tau_{2Theoretical} := \frac{15}{v_{prop.}} = \left(4.694 \cdot 10^{-5}\right)\ s$$

$$\tau_2 := \frac{41\ \mu s + 54\ \mu s}{2} = \left(4.75 \cdot 10^{-5}\right)\ s \qquad \boxed{\tau_2 = 47.5 \cdot \mu s}$$

RESPONSE TO RECTANGULAR IMPULSE, NODE 15

- The previous slide shows 2 separate oscilloscope captures of the input signal and the voltage signal across node 8.
- As was done for the signal at node 7, the time-delay between the two signals was averaged between the two captures, which led to the experimental node 15 value for Tau.
- The theoretical value of Tau was calculated in the same way as before: Using the ideal values of inductance and capacitance, calculating the wave velocity as :$(1/\sqrt{C * L})$, and then dividing the distance of the node (15) to find theoretical Tau.
- As can be seen in the previous slide, the experimental and theoretical Tau were once again close, showing an accurate analysis once again.
- The percent error between theoretical and experimental Tau was +1.193%

FREQUENCY RESPONSE AND CUT-OFF FREQUENCY OF TRANSMISSION LINE

- The final part of the analysis of the transmission line was to experimentally approximate the cut-off frequency of the TL.
- I did so by applying a 1V sine input signal at varying frequencies, between 10 to 120 kHz, with a matched load of 50Ω.
- The RMS amplitude of the wave was measured at the final node of the transmission line for each frequency, tabulated on this slide.
- By taking the common convention that the cut-off frequency is where the amplitude of the output signal is $\sqrt{2}/2$ the amplitude of the input signal, I first approximated the cut-off frequency to be 70 and 80 kHz.
- The theoretical cut-off frequency calculation is provided here.

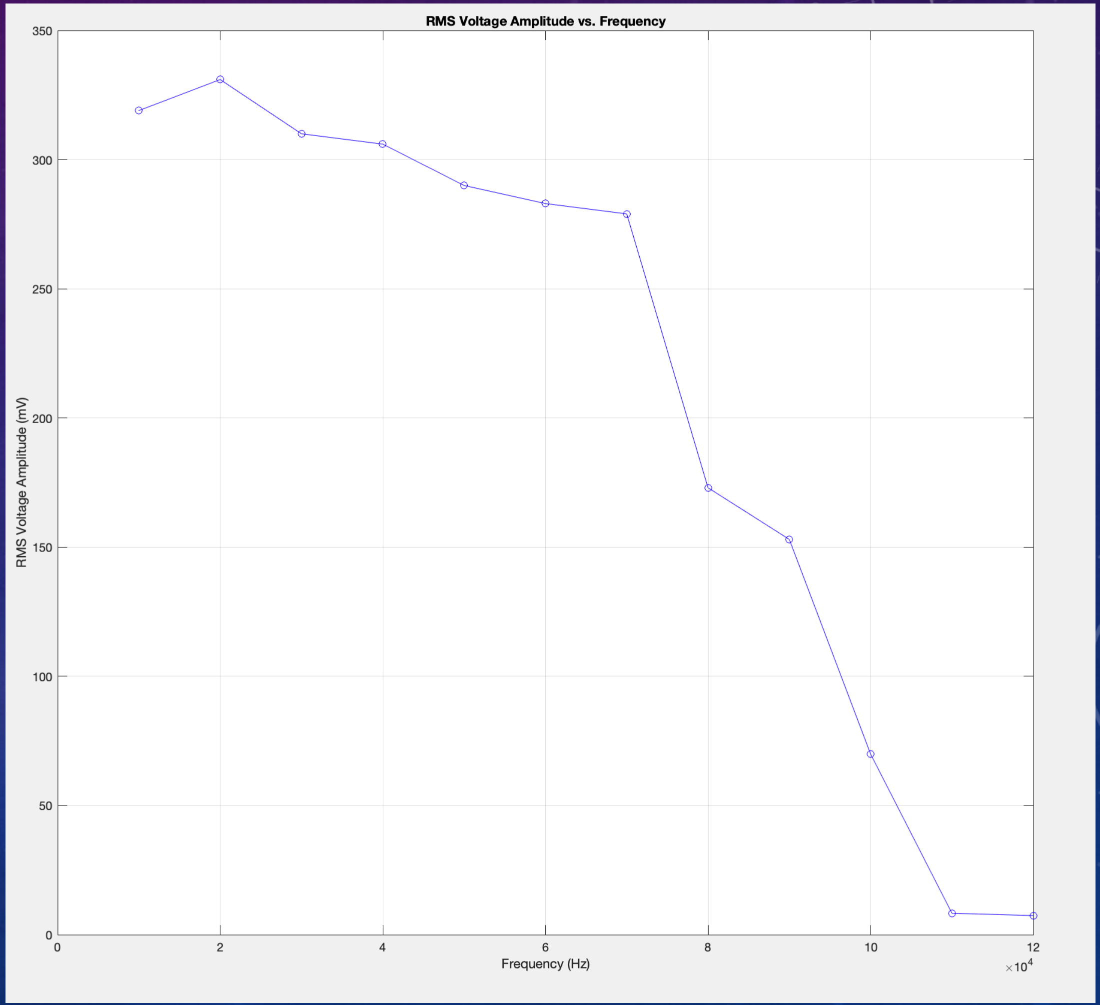
Input Frequency of 1V Sine Input:	RMS Voltage Amplitude at Node 15
10kHz	319mV
20kHz	331mV
30kHz	310mV
40kHz	306mV
50kHz	290mV
60kHz	283mV
70kHz	279mV
80kHz	173mV
90kHz	153mV
100kHz	69.9mV
110kHz	8.3mV (noisy)
120kHz	7.4mV (very noisy!!)

Theoretical Cutoff Frequency:

$$f_c := \frac{1}{\left(2 \cdot \pi \cdot (L \cdot C)^{\frac{1}{2}}\right)} = \left(5.086 \cdot 10^4\right) \frac{1}{s}$$

FREQUENCY RESPONSE AND CUT-OFF FREQUENCY OF TRANSMISSION LINE

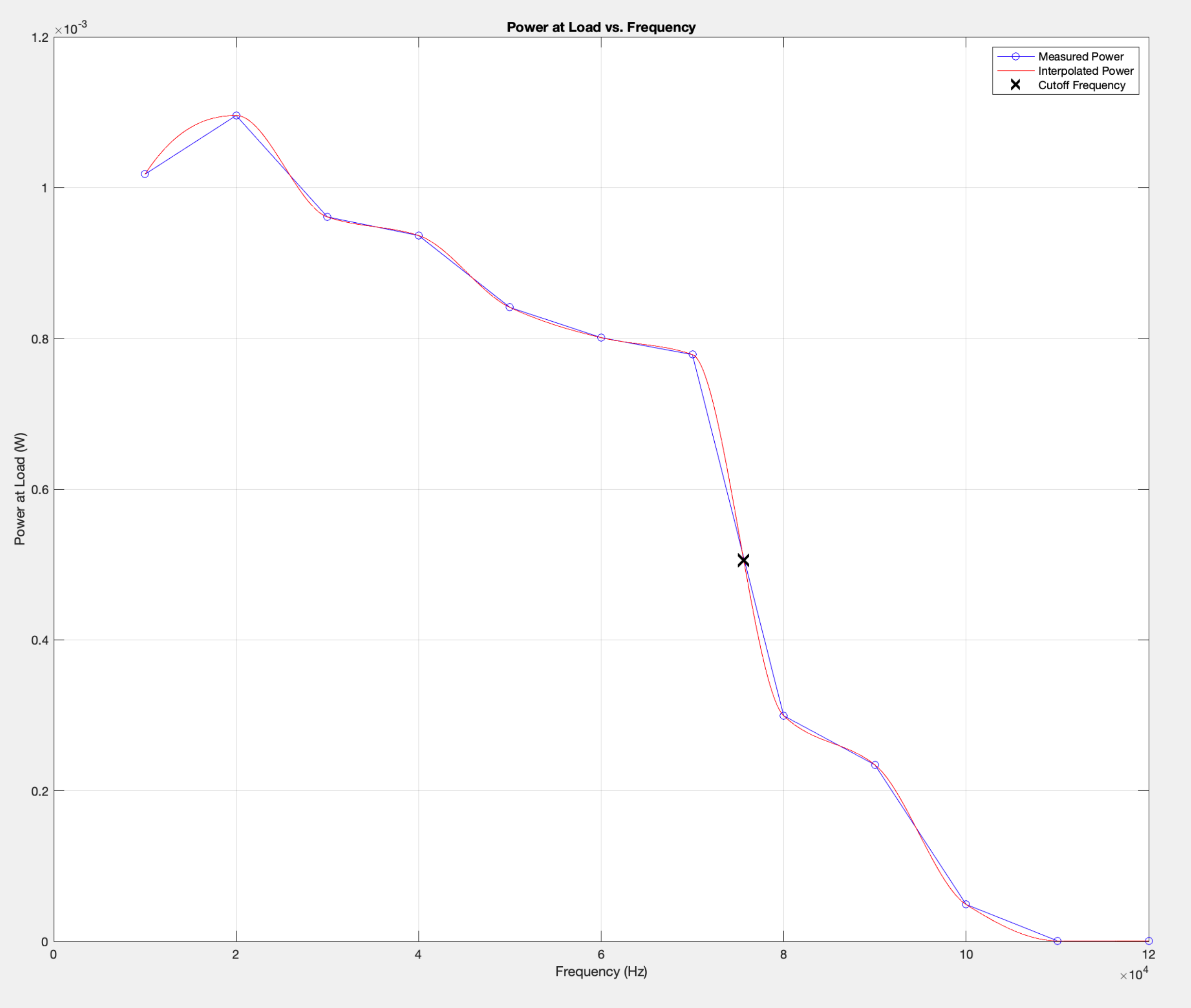
This graph shows the amplitude of the signal at node 15 vs. the input frequency.



CUT-OFF FREQUENCY OF TRANSMISSION LINE, POWER CALCULATION:

By utilizing the relationship:
$PL=|V15|^2/(2ZL)$, plotting the power at node 15 at each frequency, and interpolating a continuous function from our discrete graph (using built-in Matlab function 'interp1'), I could more accurately approximate the cut-off frequency.
The frequency value at which the output signal of node 15 carried 50% of the power of the input signal was found to be 75,625.63 Hertz.

The experimental cut-off frequency appears to have a substantial margin of error with the theoretical cut-off frequency (48%!).

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>> PowerCalculation
The cutoff frequency is approximately 75625.63 Hz
>>
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CONCLUSION:

- This project successfully achieved its primary objective of constructing an artificial transmission line and meticulously measuring its key properties. The experimental approach taken in this project allowed for a hands-on application of the knowledge of transmission lines provided by this course, a key topic in electromagnetic theory we have emphasized throughout this semester.
- Through careful measurements and analysis, the project allowed me to better understand many of the characteristics and key values that define the propagation of a wave through a transmission line: wavelength, SWR, reflection coefficient, and changes in phase along the conductive path.
- I was also able to successfully measure the response of the transmission line to a square impulse, and experimentally measure the circuit's cut-off frequency.
- This project was elucidating and enjoyable and taught me a lot about the practical implications and limitations of transmission lines.
- Thank you!