ECE 4331:

Linear Systems & Signals Laboratory

Experiment 6

Active Butterworth Low-Pass Filters

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I have neither received nor provided any assistance on this work.

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I: Introduction:

In the world of signal processing and electronics, the design and application of filters play an important role in shaping and controlling signal characteristics. The purpose and objectives of ECE 4331 Experiment 6 focused on the realization and analysis of a fourth-order Butterworth low-pass filter. It was an insightful exercise into the experimental practice of filter design. This experiment provided us with a practical understanding of filter design, beyond the theory we have studied thus far in class.

The Butterworth filter is known for its maximally flat frequency response in the pass-band region and is an ideal example to study the nuances of filter design. By focusing on a low-pass filter, this experiment delved into the essential functionality of allowing signals with frequencies below a certain cutoff frequency to pass through while attenuating those above it. Such filters are crucial in numerous applications, from audio processing to noise reduction in communication systems, as has been taught to us ECE 4330.

Through this experiment, we sought to verify the theoretical principles discussed in ECE 4330 but also to gain hands-on experience in the practical implications of filter implementation. This included understanding the selection of appropriate components, and the analysis of the filter's frequency, step, and impulse responses in a laboratory setting.

II: Procedure:

II.a: Components Used:

In the completion of experiment 6, the following components and laboratory tools were used:

- *RIGOL DG1022* Waveform Generator
- RIGOL MSO1104Z Oscilloscope
- RIGOL DM3058 digital multimeter
- LF351 operational amplifier (3)
- 18kΩ (1/4 Watt, 5%) resistor (2)
- 20kΩ (1/4 Watt, 1%) resistor (1)
- 43kΩ (1/4 Watt, 5%) resistor (2)
- 150pF (Polypropylene, 1%) capacitor (1)
- 820pF (Polypropylene, 1%) capacitor (1)
- 1nF (Polypropylene, 1%) capacitor (2)
- 0.01μF (Polypropylene, 5%) capacitor

II.b: Pre-Laboratory Procedure:

The following steps described in this subsection were completed in the fulfillment of the pre-laboratory activity for experiment 6:

a. Using the AFD Toolbox software in MATLAB, generate all of the simulation results (plots), shown in the background section of the lab instructions, for the 4th order Butterworth low-pass filter with $|H(0)| = 1$ and $fc = 10$kHz for:
   - ideal components
   - actual (5% tolerance) components values.

b. Use Mathcad to compare the magnitude response ($|H(f)|\ vs\ f$) of the two filters from Step a. Employ a logarithmic scale for frequency. Recall that $\omega = 2\pi f$ and that each stage has a transfer function given by:

$$H(f) = \frac{\frac{1}{R_1R_2C_1C_2}}{(j2\pi f)^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right)(j2\pi f) + \frac{1}{R_1R_2C_1C_2}}$$

c. Determine the cutoff frequencies of the ideal, and the actual (5% tolerance components) filters.

d. Employ Multisim to verify the frequency response of the filter you designed in Step a. (use the 5% tolerance components). Also, determine the response of the filter to a 1kHz square signal. Repeat for a 10kHz frequency.

II.c: Laboratory Activity Procedure:

The following steps described in this subsection were completed during the in-person laboratory session for the fulfillment of the laboratory activity:

a. Use the breadboard to build the 4th order low-pass Butterworth filter that you have designed and analyzed in the Pre-lab activity. Employ two LF351 op-amps.

b. Connect the input of your filter to the waveform generator. Apply a sine wave with 2Vpp. Monitor the input signal and the filter's output signal on CH1 and CH2 of the oscilloscope, respectively. Make sure the scope channels are set to "INVERSION: OFF". Determine, experimentally, the input sinusoid (cutoff) frequency that leads to about 0.7 Volt amplitude for the output signal. Capture and save the scope's display image. The image should display in the measurements bar the following values: input signal frequency, input signal max voltage, output signal max voltage and the phase difference between the output and the input.

c. Tabulate the output signal amplitude and phase for the following input frequencies: 1kHz, 2kHz, 3kHz, ..., 9kHz, 10kHz.

d. Change the input signal to a 2Vpp, 50% duty cycle square wave. Determine (and save the scope's display image) the filter's output for the following square wave frequencies: 2kHz, 3kHz, 4kHz, 5kHz and 10kHz.

e. Determine the step-response of your filter. Set the input to a 50% duty cycle, 1kHz square wave with low level at 0Volt and high level at 1 Volt. Set the scope to "averaging mode". Save the scope's display image. Next, press the CURSOR button and set the mode to "Track" and set Cursor A to CH2. Use the "INTENSITY" knob to position the cursor at the peak of the filter output. Save the scope's display image.

f. Determine the impulse response of the filter. Keep the same settings as in Step e but change CH1 and CH2 to 1V/div. Connect the output of the filter to the input of the differentiator circuit, shown below. Connect the output of the differentiator to CH2. Set CH2 to "Invert: ON" mode (this is necessary to undo the inversion inherent in the differentiator). The idea here is to generate the impulse response of the filter as the derivative of the step-response. Move the cursor so that it reads the time and max amplitude of the impulse response. Measure the minimum amplitude of the impulse response and the time it occurs at. You will be comparing these values to those you generated from the AFD simulation from the Pre-lab. Save the scope's display image.

g. Use the AnalogDiscovery2 (AD2) and the WaveForms software on your PC to generate the frequency response of your filter. Connect the output of the filter to CH2 (scope input 2) of the AD2. Connect the AD2 module to your laptop (via the supplied USB cable) and run the provided WaveForms software and program file for experiment 6.

III: Results:

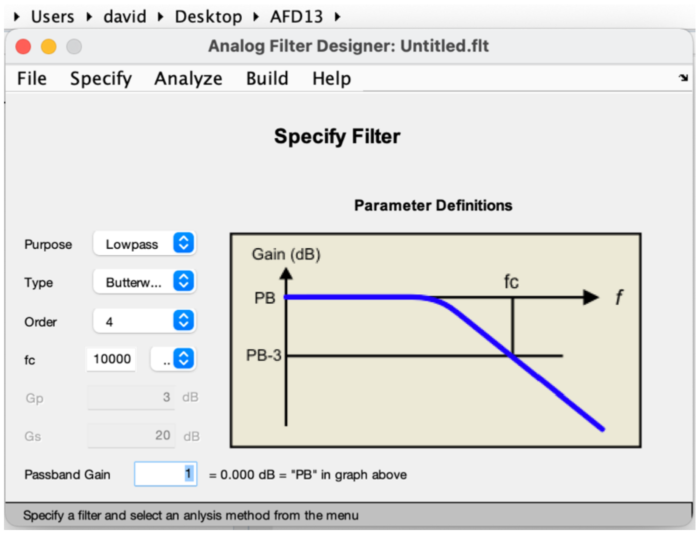
The following sections describe and analyze in detail the theoretical and experimental results obtained during the completion of the pre-laboratory and the laboratory activity. Many of the circuit elements' theoretical vs. experimental values, and their relative error, are tabulated in these sections. Additionally, certain key findings and values are highlighted and boxed.

III.a: Pre-Laboratory Results:

Step A: Simulating 4^{th}-Order Low-Pass Butterworth Filter using AFD Toolbox in MATLAB:

The following 8 images demonstrate the simulated results of using the provided AFD MATLAB Toolbox to create and analyze various parameters of a 4^{th}-order Butterworth LPF with a passband gain of 1V/V and cut-off frequency of 10kHz.

Figure 1: Initial Parameters Chosen to Create 4th-Order Butterworth LPF



As can be seen in the image above, the initial parameters specified for the version of our circuit simulated circuit were correctly selected in the landing page of the ADF Toolbox.

Figure 2: Transfer Function of Simulated Butterworth LPF

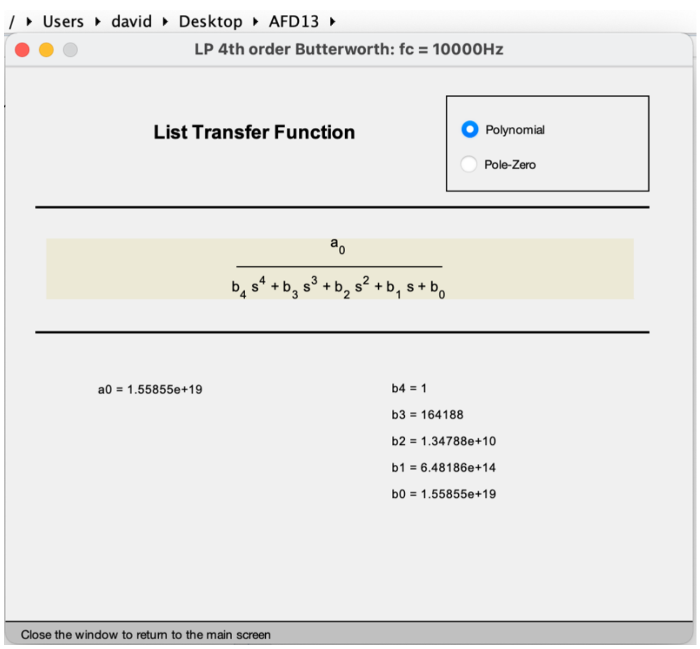


Figure 2 above shows the transfer function of the idealized 4th-order Butterworth LPF. Figure 3 and 4 show the circuit model of the 4th-order Butterworth LPF we are attempting to simulate. The 4th-order circuit is achieved by cascading two separate 2nd-order Sallen-Key circuits. Both Sallen-Key circuits were set to have 5% component tolerances.

Figure 3: Sallen-Key 1 Specifications

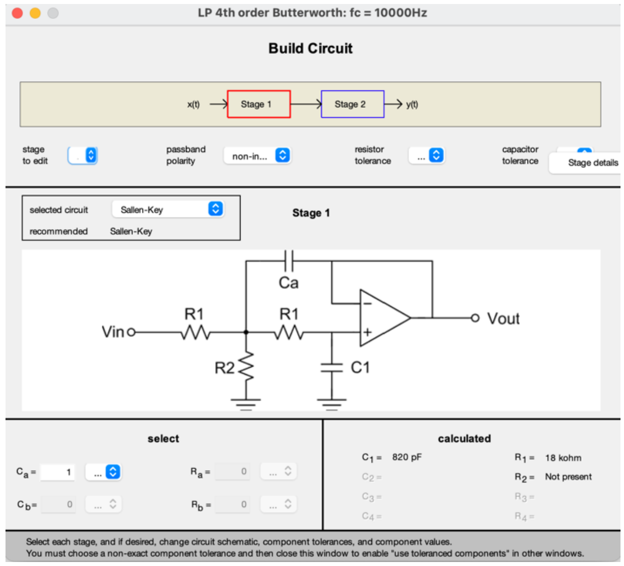


Figure 4: Sallen-Key 2 Specifications

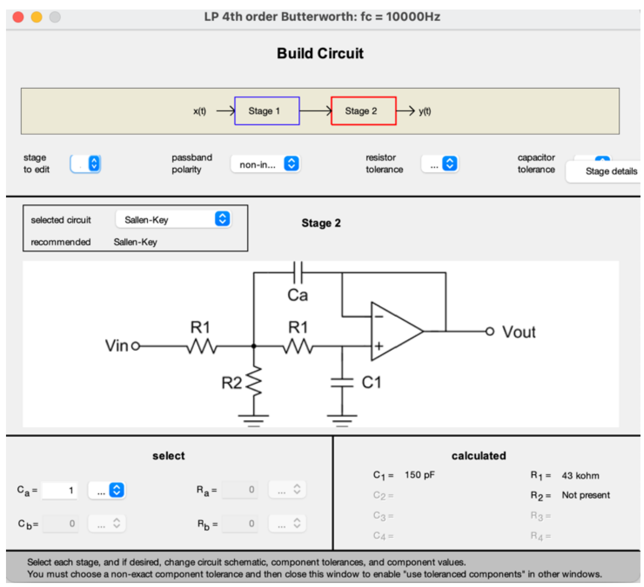


Figure 5 below shows the graph of the poles of the transfer function of our simulated 4th-order Butterworth LPF. There are two sets of poles: 4 poles for the idealized LPF (black x's) and 4 poles for the more 'practical' LPF containing circuit elements with 5% tolerances (blues x's). As is to be expected, the 4 poles all lie on the left-hand side of the complex plane, indicative of a stable system.

Figure 5: Location of Poles of H(s) of 4th-Order LPF

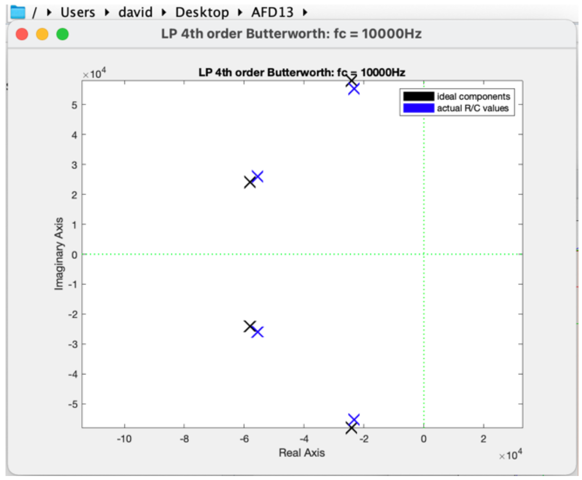
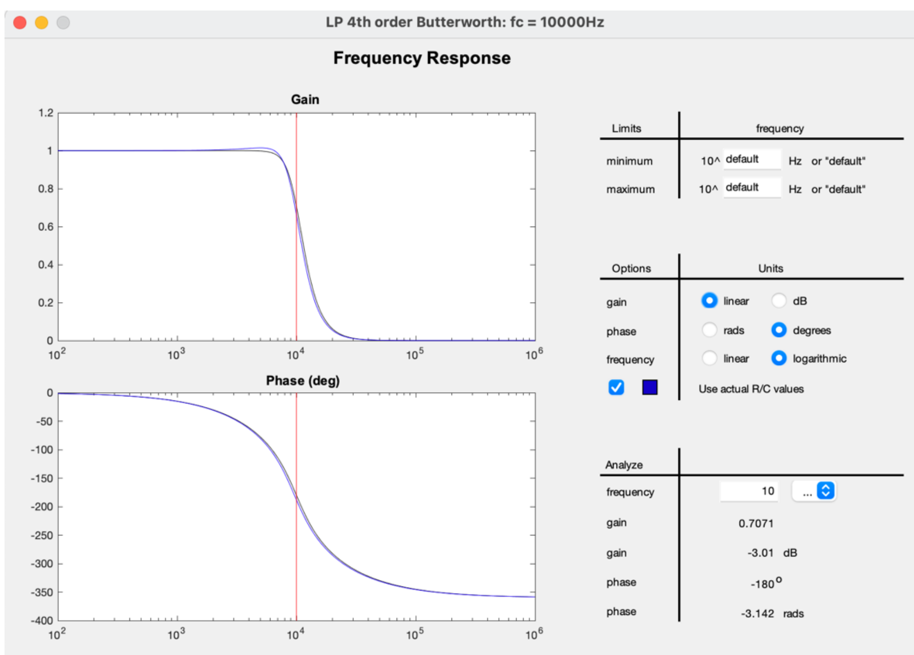


Figure 6 below shows the simulated frequency and phase response of both the idealized and 'practical' (5% tolerance) 4th-order LPFs. The responses of both simulated filters are very close to one another: As can be seen in the later sections of this lab report, these results very closely reflect the experimental results obtained. Since the circuit components used in the experimental realization of the system were all well the below 5% error margin, the experimental results lie between the two results shown in figure 6.

Figure 6: Simulated Frequency and Phase Response of 4th-order LPF



Figures 7 and 8 show the simulated results of the Unit-Step and Unit-Impulse response of idealized and 5% tolerance 4th-order Butterworth LPF. These results will be particularly important when compared to the experimental results obtained in steps E and F on the laboratory activity and will provide valuable insights into both the accuracy and limitations of the experimental system and results.

Figure 7: Simulated Unit-Step Response of Butterworth LPF

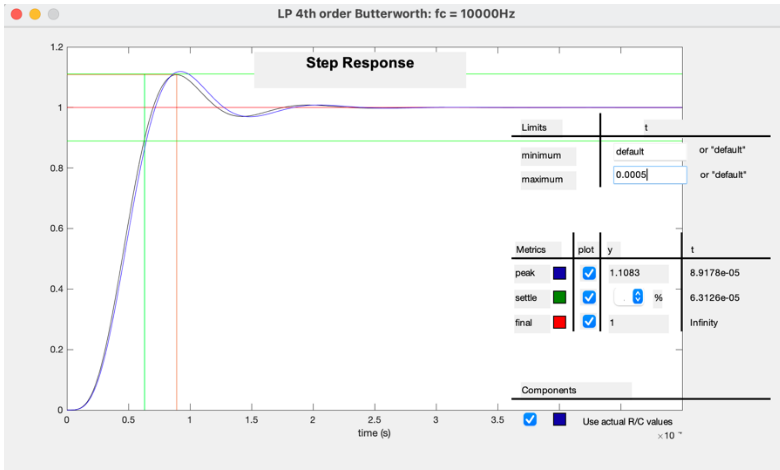
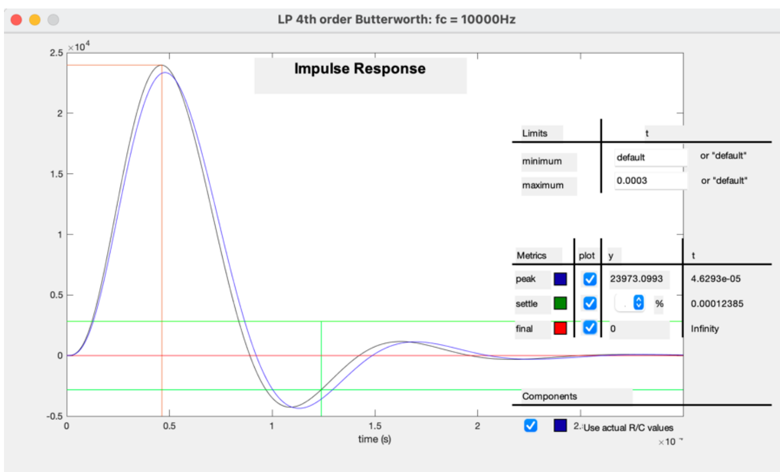


Figure 8: Simulated Unit-Impulse Response of Butterworth LPF



<u>Step B: Mathcad Simulations of 4th-order Butterworth LPF:</u>

<u>Step C: Estimating Cut-Off Frequency of Simulated Butterworth LPFs:</u>

Figures 9 and 10 show 2 differently scaled views of the same frequency graph. Unfortunately, the AFD Toolbox does not have flexible tracing capabilities for newer version of MATLAB. We know, however, that the ideal filter has been set to have a cut-off frequency of exactly 10kHz. Meanwhile, the more 'practical' simulated circuit with 5% tolerances in its circuit elements (blue lines) show that this small margin of error pushes the transfer function and therefore the system's cut-off frequency slightly below 10kHz, to approximately 9.5kHz.

Figure 9: Frequency Response of Simulated Filters, 1

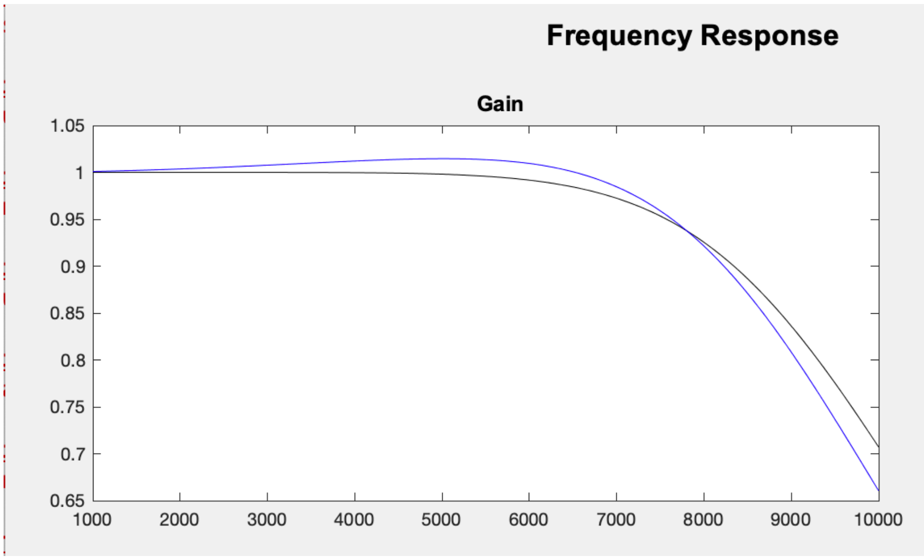
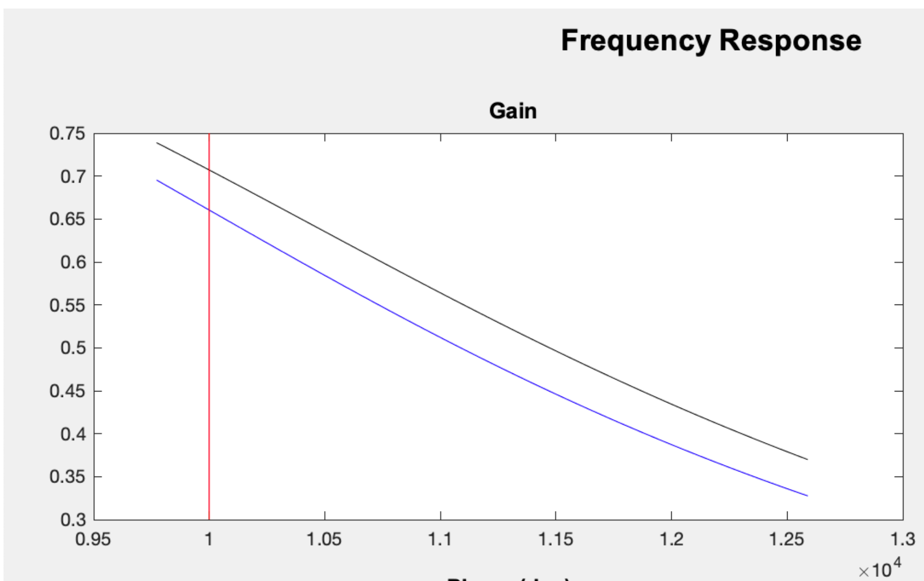


Figure 10: Frequency Response of Simulated Filters, 2



<u>Step D: Multisim Simulations</u>

The following 4 figures show the results of simulating the 4th-order Butterworth LPF, at 5% tolerances, using NI Multisim. Figure 11 shows the circuit implementation we will base our experimental implementation on. Figure 12 shows the system's input and output signals when the input signal is set to a 2Vpp sinusoid at 10 kHz, which will provide a useful reference for our results in steps B and C of our laboratory activity. Figure 13 shows the frequency response of the Multisim simulation, which will also provide a useful reference, and coincides with the values we obtained in the AFD simulations: The Multisim results show a cut-off frequency of approximately 10kHz, similar to the AFD results. Figures 14 and 15 show the simulated system's response to a square wave at frequencies of 1kHz and 10 kHz, respectively, which provide us with a reference point for the experimental results we will obtain in step D of the laboratory activity.

Figure 11: Multisim Simulated Design of 4th-Order Butterworth LPF

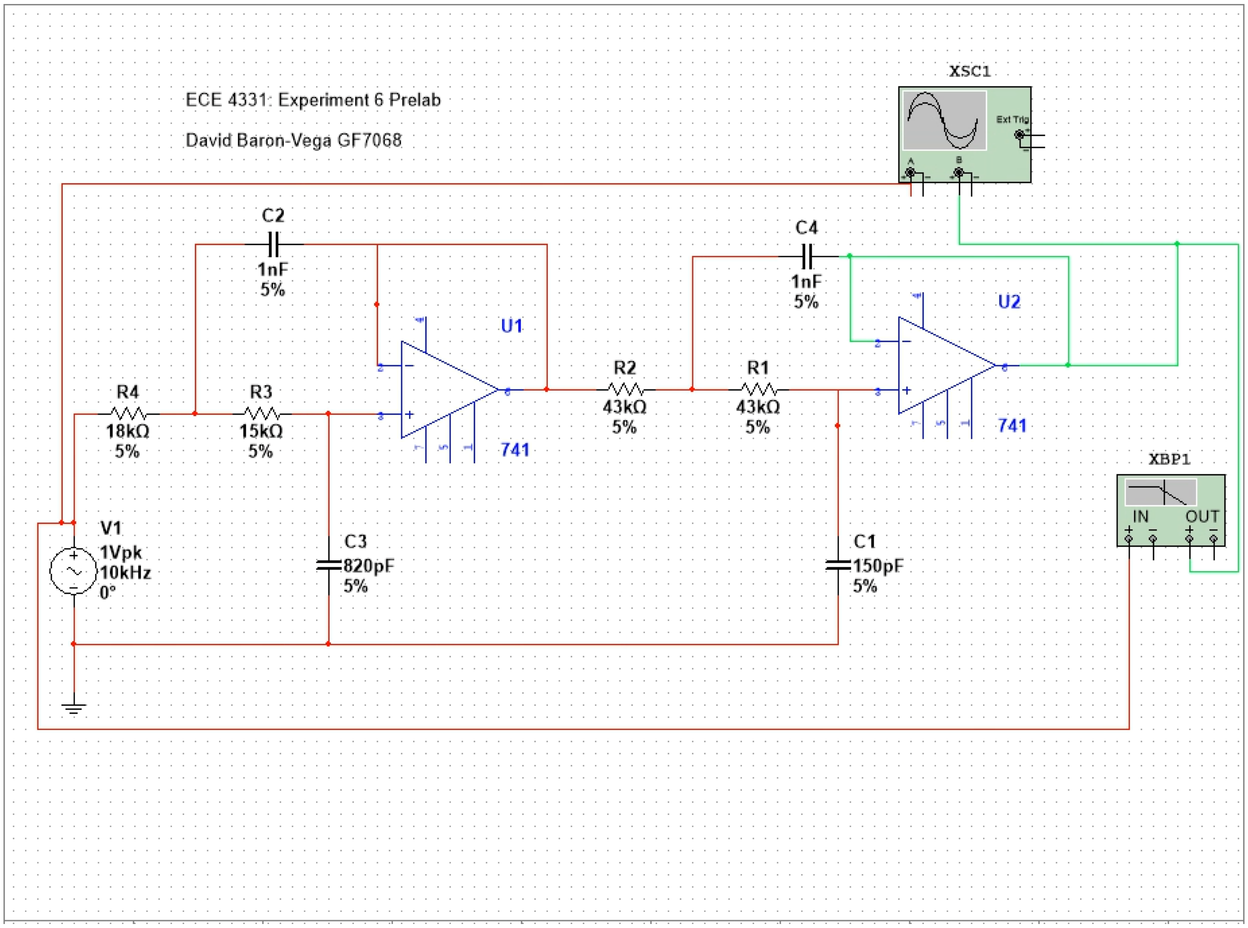


Figure 12: Simulation Results of Filter's Response to 10kHz Sinusoidal Input

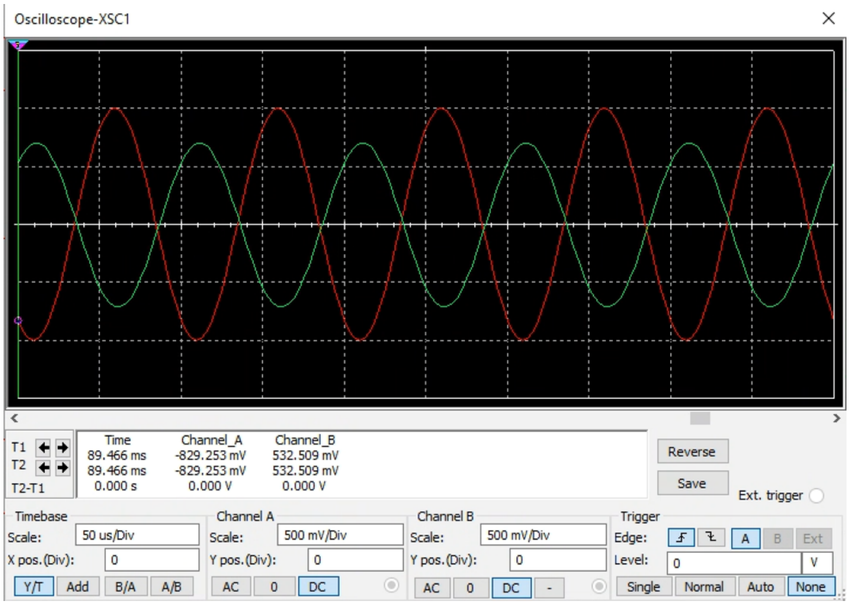


Figure 13: Simulated System's Frequency Response

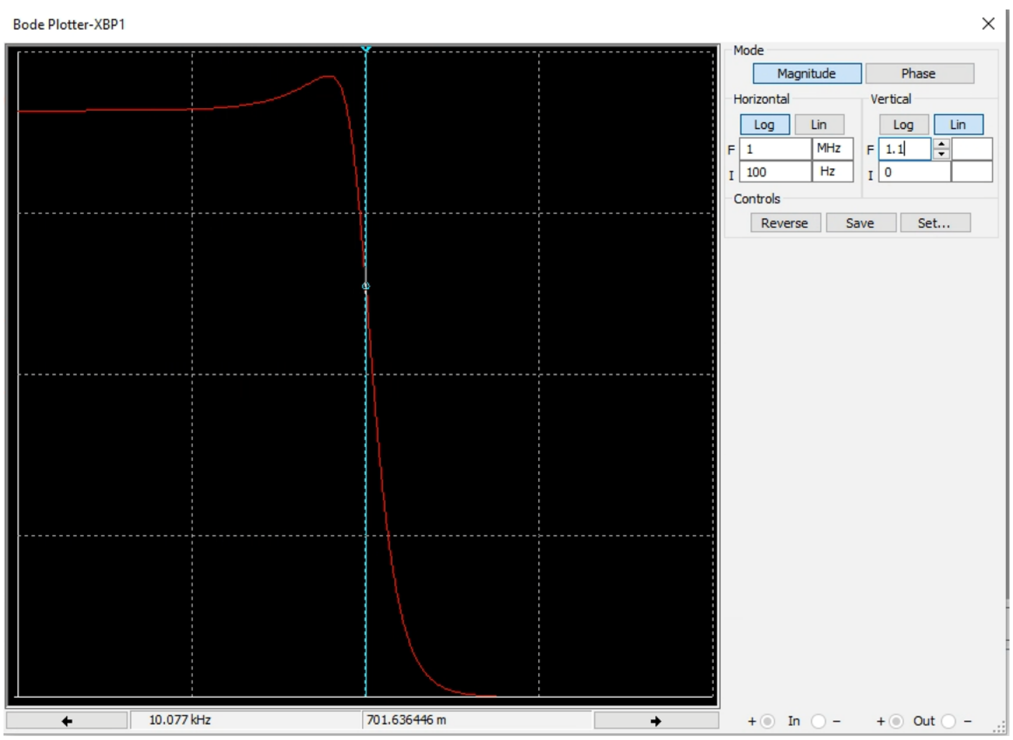


Figure 14: Simulated System's Response to 1kHz Square-Wave Input

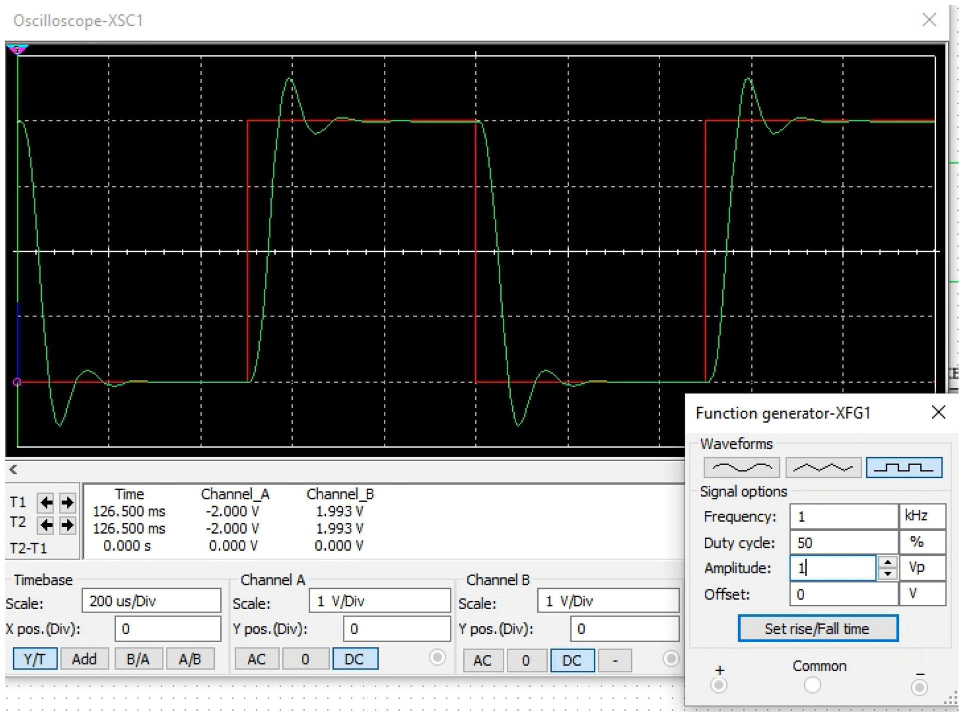
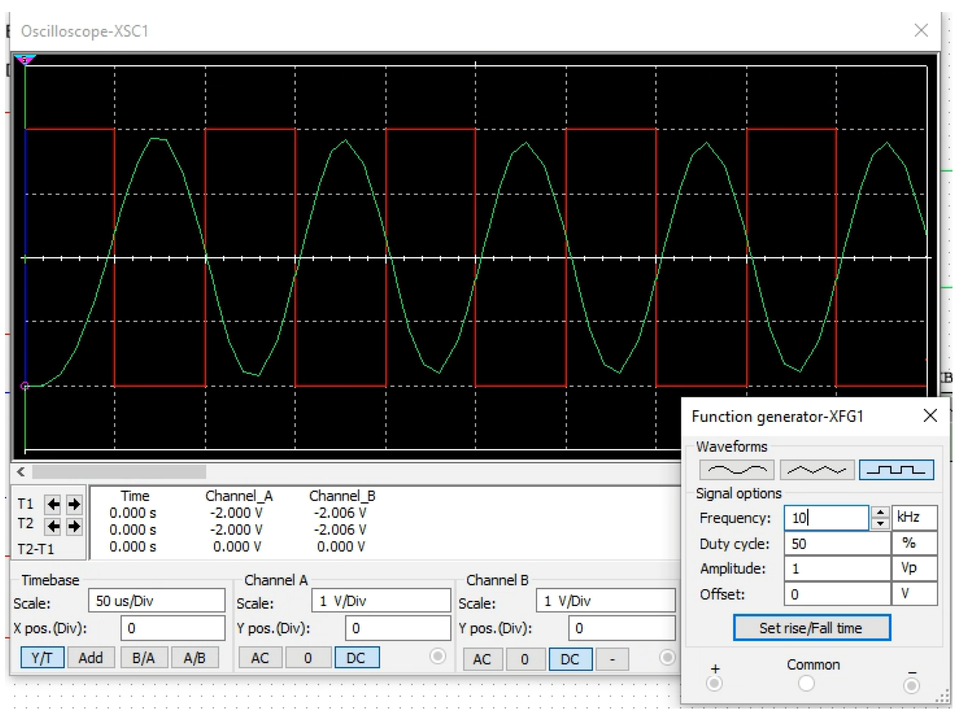


Figure 15: Simulated System's Response to 10kHz Square-Wave Input



III:b Laboratory Activity Results:

This section provides all the experimental results obtained during the completion of the laboratory activity, in person.

Table 1 below provides the experimental vs. theoretical values, and the relative error, of all the circuit components used in the completion of each of the proceeding steps.

Table 1: Percent Error of All Circuit Components Used

Component Name:	Theoretical Value:	Experimental Value:	Percent Error:
C1 (1st Sallen Key)	1nF	999pF	-0.1%
C2 (2nd Sallen Key)	1nF	1001pF	0.1%
C3 (1st Sallen Key)	820pF	819pF	-0.122%
C4 (2nd Sallen Key)	150pF	145pF	-3.33%
C5 (Differentiator)	10nF	9.73nF	-2.7%
R1 (1st Sallen Key)	18kΩ	18.05kΩ	0.277%
R2 (1st Sallen Key)	18kΩ	17.85kΩ	-0.833%
R3 (2nd Sallen Key)	43kΩ	43.1kΩ	0.233%
R4 (2nd Sallen Key)	43kΩ	42.826kΩ	-0.405%
R5 (Differentiator)	20kΩ	19.947kΩ	-0.265%

Because we are carefully considering how deviations from ideal component values can alter the resulting transfer function of the Butterworth filter we have simulated and created, it is worth noting that the experimental percent error of all of the circuit components used was found to be very low, all well below the 5% threshold we have simulated thoroughly in the pre-lab. Although other sources of error, such as noise in function generator and any resistance and noise brought upon by the oscilloscope and measuring tools, we should expect our experimental results to be between the thresholds of the simulated values of the ideal and 5% error filters.

Step A: Circuit Realization:

The two figures below demonstrate the physical realization of the 4th-order Butterworth low-pass filter created in lab and used for the experimental analysis of experiment 6. The only discrepancy between the first and second figure is the addition of a cascaded differentiator circuit, created using an additional LF351 op-amp. The second circuit was used to experimentally measure the system's impulse response, by differentiating the system's step-response, in step F.

Figure 16: 4th-order Butterworth LPF Circuit Realization

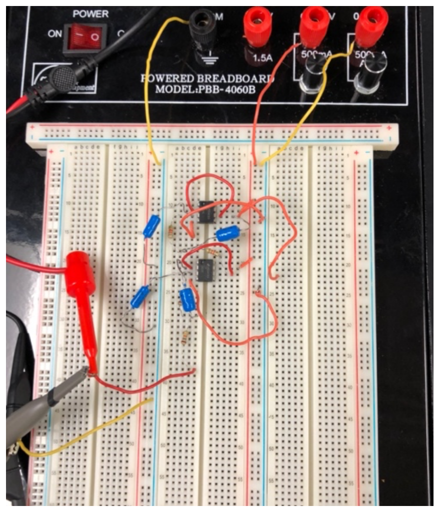
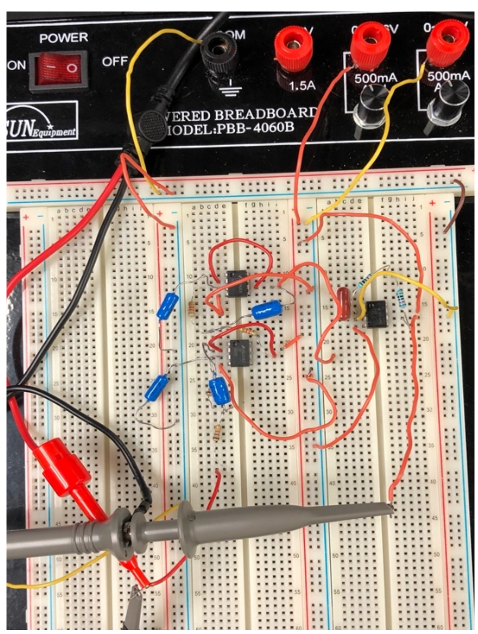


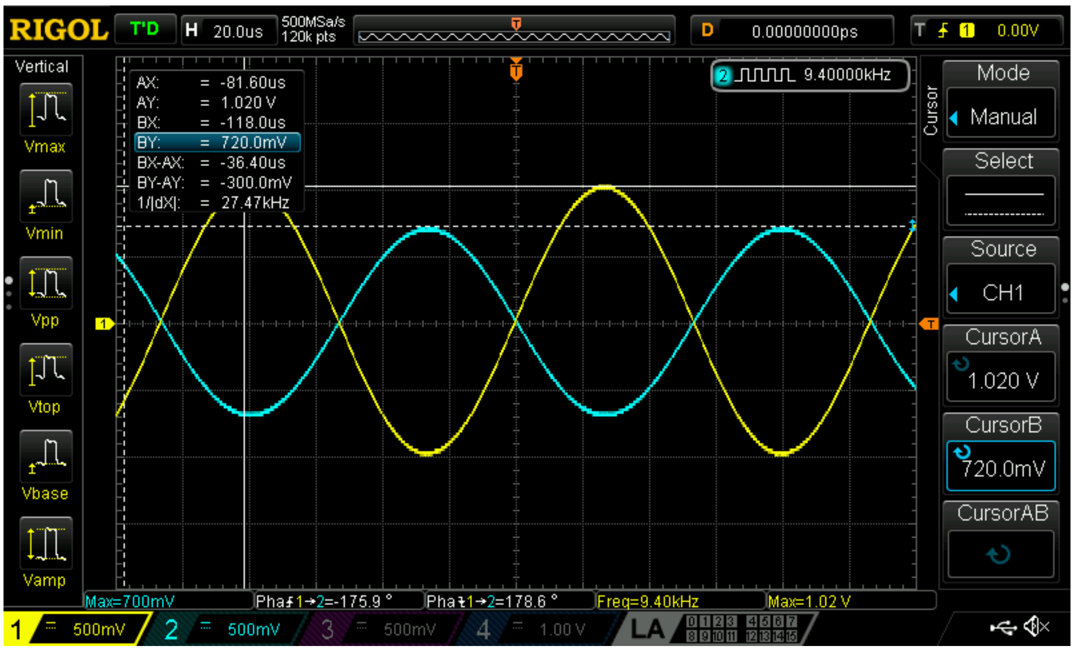
Figure 17: 4th-order Butterworth LPF Circuit Realization w/ Cascaded Differentiator



Step B: Experimental Cut-Off Frequency:

In step B, a 2Vpp (1V amplitude) sinusoidal signal was applied at the input of the Butterworth filter. The input signal (yellow) and output signal (blue) were measured using the oscilloscope, and the input frequency was gradually increased until the output signal reached an amplification factor of approximately Vin x 0.7071, which indicates the point of cut-off. The experimental cut-off frequency of our circuit was measured at 9.4 kHz, and the phase-difference between the two signals was approximately 177.25 degrees. These experimental results demonstrate a high degree of accuracy with respect to our simulated results.

Figure 18: Measured Cut-Off Frequency of System



Step C: Tabulation of Output Amplitude and Phase Difference as a function of Input Frequency

In step C, we observed the behavior of the Butterworth filter as we varied the input signal's frequency. These results show the filter behaving as expected, and very accurately: The amplitude of the output signal remains very stable within the pass-band region of the filter, and we see its phase-shift increasing linearly. We observe the LPF's 'Knee-Frequency' (i.e., the frequency where the signal begins to be attenuated) at approximately 7kHz. Once again, we observe the cut-off frequency around 9.4kHz, and witness a step attenuation from that point forth. Once again, these results correlate strongly with the simulated results obtained during the pre-lab activity.
Oscilloscope captures of the input and output signals at each of these frequencies are included in the appendix of this lab report.

Table 2:Output Amplitude and Phase Difference as a function of Input Frequency

Input Frequency:	Output Amplitude:	Output Phase Difference
1kHz	1.00V	15.2°
2kHz	1.00V	33.1°
3kHz	1.02V	47.5°
4kHz	1.02V	67.2°
5kHz	1.02V	83.4°
6kHz	1.02V	103.3°
7kHz	0.980V	124.6°
8kHz	0.90V	148.4°
9kHz	0.760V	170.4°
9.4kHz	0.700V	177.25°
10kHz	0.620V	166.7

Step D: Response of Filter for Square-wave Input, at Varying Frequencies:

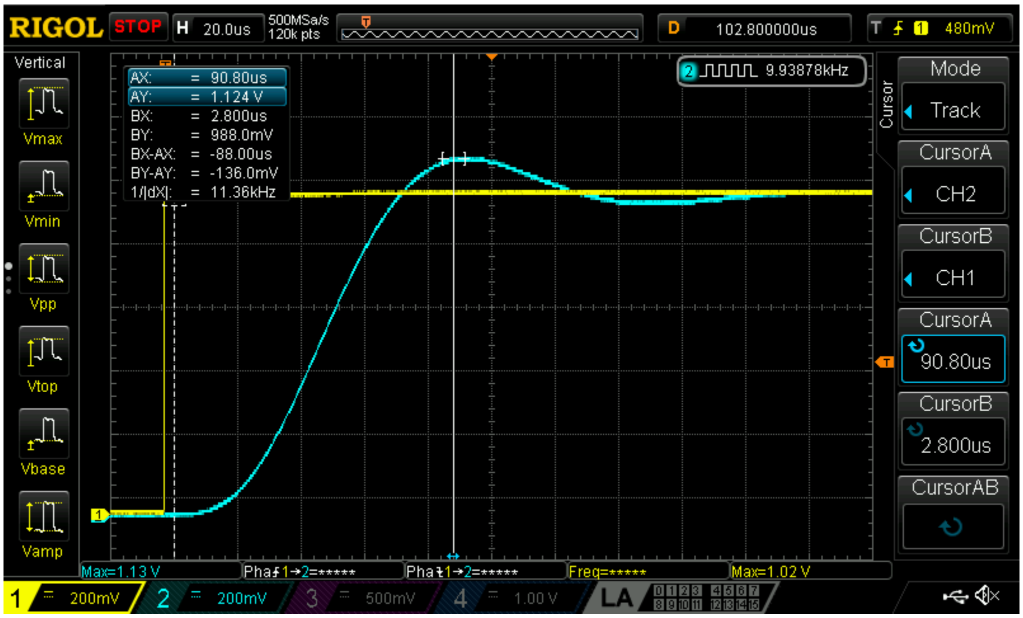
In step D, we analyzed the response of the Butterworth filter when subjected to a square wave input at varying frequencies. Initially, with a 2Vpp square wave signal at 1 kHz, the filter's output exhibited the expected transient oscillatory response, indicative of the higher harmonic frequencies inherent in the square wave being momentarily passed before attenuation. These transient oscillations show the circuit's reactive nature to the sudden changes in voltage characteristic of a square wave's leading and trailing edges. As the input frequency was increased, particularly noticeable at 4kHz and beyond, we began to witness a diminishing of the oscillatory transient jumps, with the filter's progressive damping effect becoming more and more pronounced. These results are consistent with the filter's low-pass characteristics, which attenuate higher frequency components more aggressively, smoothing out the signal. By the time the input frequency reached 10 kHz, the output was extremely attenuated: all the high-frequency components/jumps of the signal's transient response vanished, and the output signal's overall amplitude was significantly attenuated. This transformation is attributed to the filter's effective suppression of the higher harmonics that construct the sharp features of a square wave, which caused the rounding strong off of the edges, leading to output signal to look like a simple sinusoid.

The experimental observations align closely with the simulated results, reinforcing the Butterworth filter's known behavior. The filter maintains output signal amplitude within the passband region. At and beyond the cutoff frequency, around 9.4 kHz, a significant attenuation is witnessed, conforming to the filter's design parameters. This concurs with the simulated results from the pre-lab and validate the Butterworth filter's predictable response to frequency variations.

Detailed oscilloscope captures supporting these findings are included in the appendix for further reference.

Step E: Unit-Step Response of the Butterworth Filter

Figure 19: Unit-Step Response of 4th-Order Butterworth Filter



In Step E, we analyzed the unit-step response of our Butterworth low-pass filter. The experimental step response demonstrated the filter's transient behavior as it responded to a sudden change. The output quickly rose and exhibited a slight overshoot before stabilizing, characteristic of the Butterworth filter's maximal flatness in the passband. Our experimental results obtained are shown in the figure above.

The simulated unit-step response, generated by the AFD toolkit provided to us, displayed very similar behavior, with the step response rising sharply at the onset and then exhibiting a similar overshoot. Both responses eventually settled to the steady-state value. The minimal discrepancy between the experimental response and the simulated one shows the precision of the filter's design and construction.

The peak transient amplitude reached by output signal in our experimental results was approximately 1.124V, which once again, closely aligns the results obtained in the simulation.

Step F: Unit-Impulse Response of the Butterworth Filter

For step F of the experiment, we attached an inverting differentiator op-amp circuit across the output of the Butterworth filter while keeping the input signal from step E the same. Essentially, we successfully measured the filter's unit-impulse-response by taking the derivative of its unit-step response; very cool! Note: The need to apply inversion on the oscilloscope was important due to the differentiator inverting the output signal. Taking accurate max/min measurements would've been challenging otherwise.

In examining the impulse response of filter, both the simulation and experimental results yielded similarly shaped results, but there are very noticeable and large quantitative differences. The simulated response achieved a maximum amplitude of approximately 2,300V before quickly decaying towards zero, then reaching a minimum value of approximately -0.4V, and then reaching steady-state stability. Meanwhile, the experimental impulse response, although similar in shape and behavior, reached a peak amplitude of approximately 4V, and a minimum amplitude of approximately -2V!
The very theoretical nature of the Dirac 'function' (it's actually a theoretical distribution) makes it difficult to compare and obtain experimentally. The experimental output is limited by the bandwidth limitations of the integrated circuits and other circuit components and is not ready-made function accessible through the function generator we used: this is why we needed to realize an analog differentiator to approximate the signal. It is also worth questioning the consequences of putting the differentiator at the output of the Butterworth filter, as opposed to putting it directly across the initial input signal.
The time it took for the experimental impulse response to peak, minimize, and stabilize was in the range of 180-220 microseconds; the simulated results of the impulse response took 200 microseconds. These results are actually very close. However, and once again, the difficulty of approximating the impulse-response stems more from the impossibility of approximating a function whose height that approaches infinity as its base approaches zero. In conclusion and taking all this into consideration, I believe our experimental results model the simulation as closely as the limitations of the physical system allowed us to.

Figure 20: Unit-Impulse Response of 4th-Order Butterworth Filter, pt.1

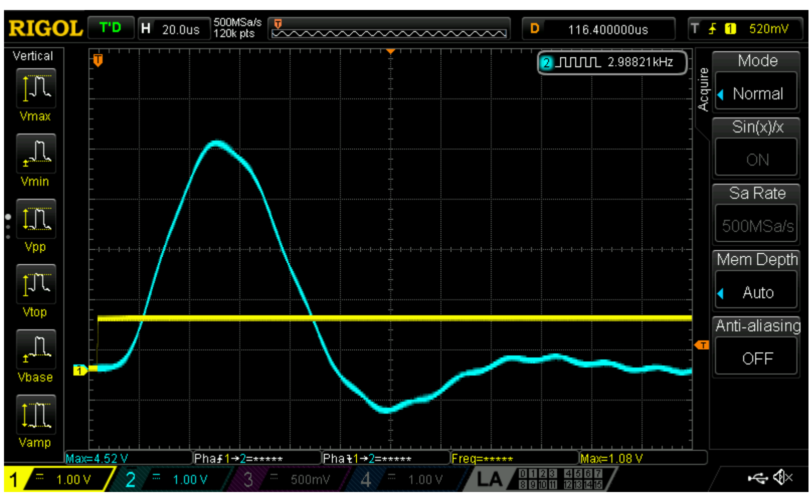
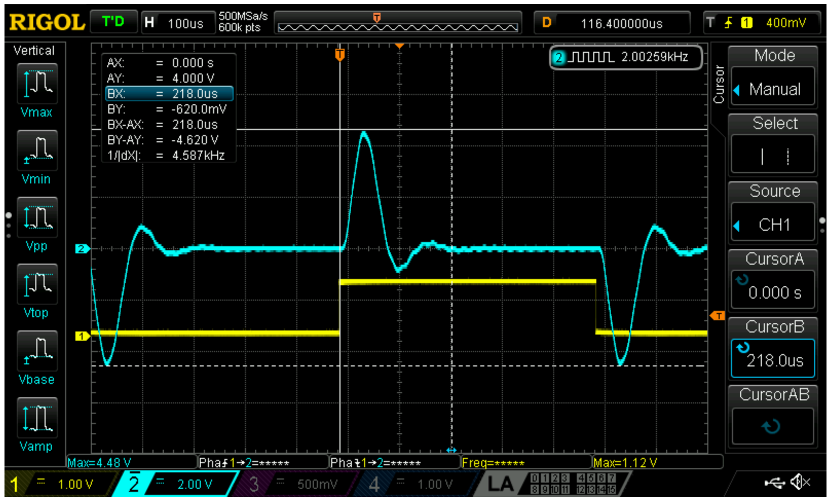


Figure 21: Unit-Impulse Response of 4th-Order Butterworth Filter, pt.2



Step G: Measuring the Experimental Cut-Off Frequency w/ AD2 & Waveforms Software:

In step G, we once again measured the experimental cut-off frequency and phase response of the Butterworth filter we constructed, but this time by utilizing the AD2 module and the accompanying Waveforms software. The resulting magnitude and phase plots of our system are provided in the figure below:

Figure 22: Magnitude, Phase Plot of 4th-Order Butterworth Circuit

As can been seen, the AD2 experimental results show the cut-off frequency of our system to be approximately 9.515 kHz, which closely parallels both the simulation results obtained in the pre-lab and the experimental results obtained in step B from having used the laboratory oscilloscope and function generator. This only a 1.22% discrepancy between these results and the results obtained in step B.

Given our previous experimental results and the simulated results, the AD2's results reaffirm our confidence in the accuracy of the previous experimental measurements obtained in all previous steps of the laboratory activity.

IV: Conclusion

In conclusion: our comprehensive analysis of Experiment 6 has provided us a detailed understanding of the design, analysis, and practical implications of a fourth-order Butterworth low-pass filter, and analog filter design, broadly speaking. The meticulous pre-laboratory simulations, employing tools like the AFD Toolbox, MATLAB, and Multisim, gave us a thorough theoretical foundation which was complemented by the tangible insights gained from our hands-on laboratory work. We generally observed accurate and precise alignment between the theoretical predictions and the experimental data, as evidenced by the close correspondence of the cut-off frequencies and the response behaviors under varying input conditions. Our experimental analysis not only corroborated the filter's theoretical attributes, such as its maximally flat passband, but also elucidated to us the nuances of real-world filter design applications, including the impact of component tolerances and the operational limitations of the test equipment and circuit components.

The successful attenuation of high-frequency components, the filter's transient response to square, step, and impulse inputs, and the general consistency of the filter's responses with those of the theoretical models, all demonstrated to us the Butterworth filter's reliability and predictability, albeit with limitations. The slight variations observed were within expected practical limits: excluded the experimental impulse response, all steps and results obtained in the experimental analysis had less than a 10% percent margin of error compared to those of the simulated models which indicates I high degree of accuracy and precision. This laboratory exercise has not only fortified our theoretical knowledge but has also honed our experimental acumen, preparing us for the intricate task of applying linear system theories to real-world signal processing tasks that we'll encounter in further courses and in the workplace.

Thank you!

ASIDE: It has been a sincere pleasure to have taken this course. I personally feel like I've learned so much, more than I expected to. I greatly enjoyed participating in this course under the instruction of Professor Kadoura and feel much more prepared for what is to come. I feel every day a bit more worthy of the title of 'engineer', and am extremely grateful for the experiences I've gained through this course, this semester. Thank you, thank you, thank you!

V: Appendix

<u>Step C Oscilloscope Results:</u>

Figure 23: Step B, 1kHz

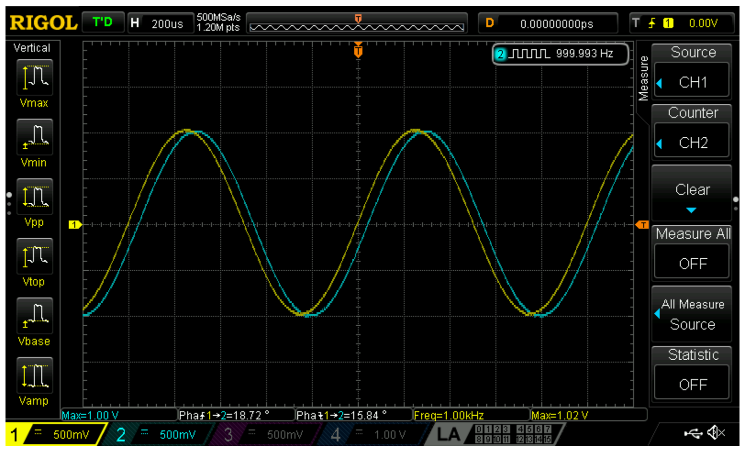


Figure 24: Step B, 2kHz

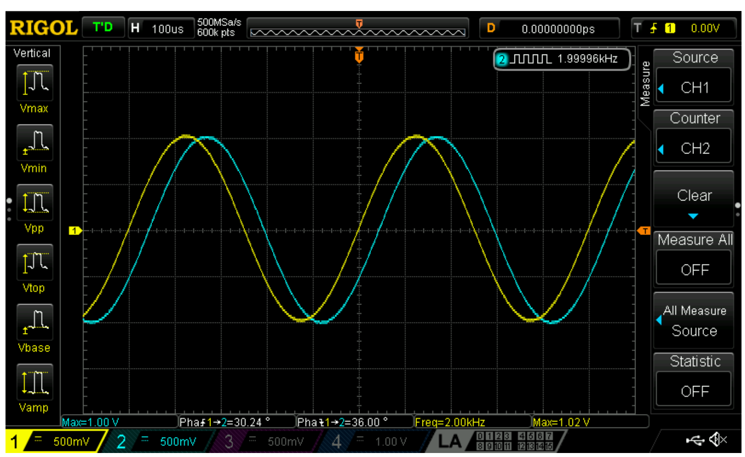


Figure 25: Step B, 3kHz

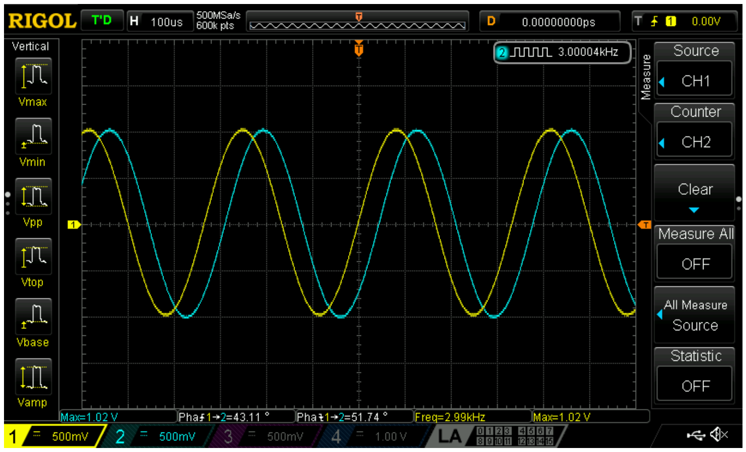


Figure 26: Step B, 4kHz

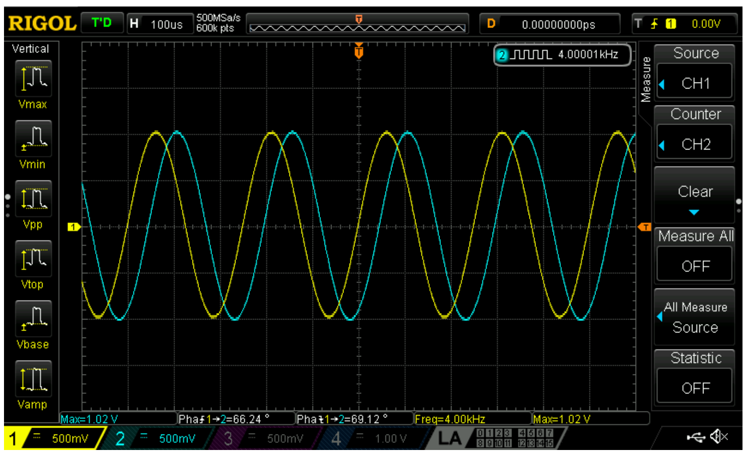


Figure 27:Step B, 5kHz

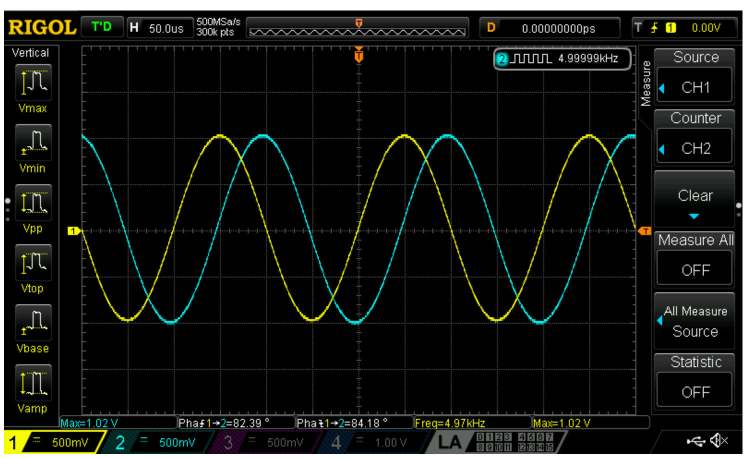


Figure 28: Step B, 6kHz

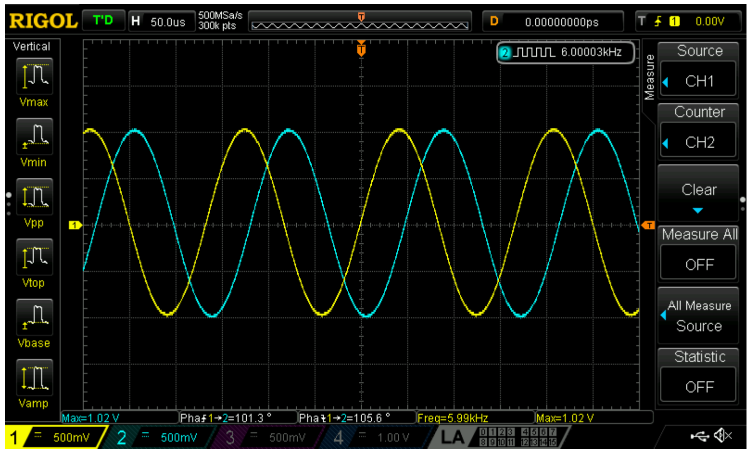


Figure 29: Step B, 7kHz

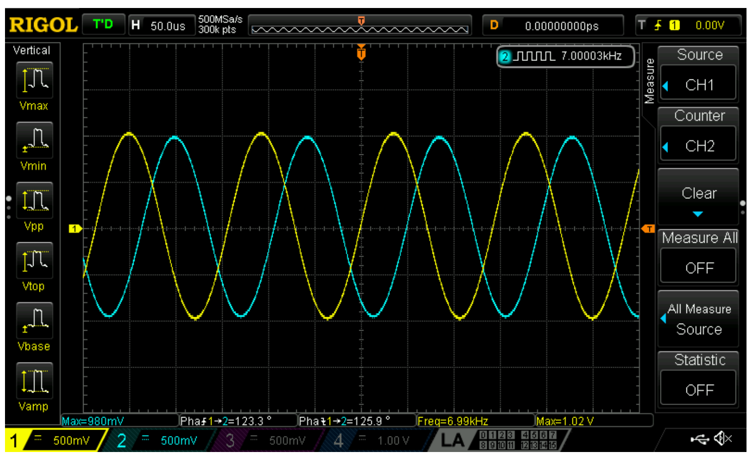


Figure 30: Step B, 8kHz

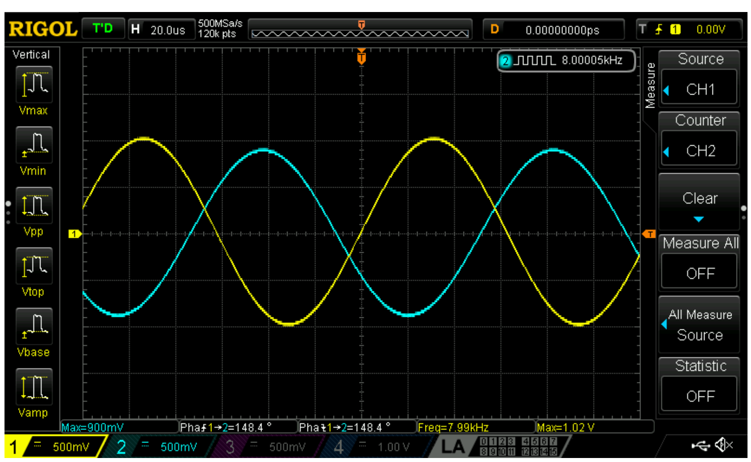


Figure 31: Step B, 9kHz

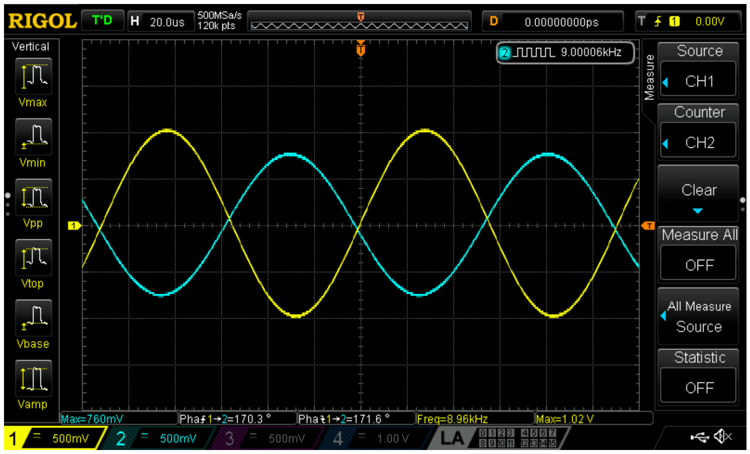
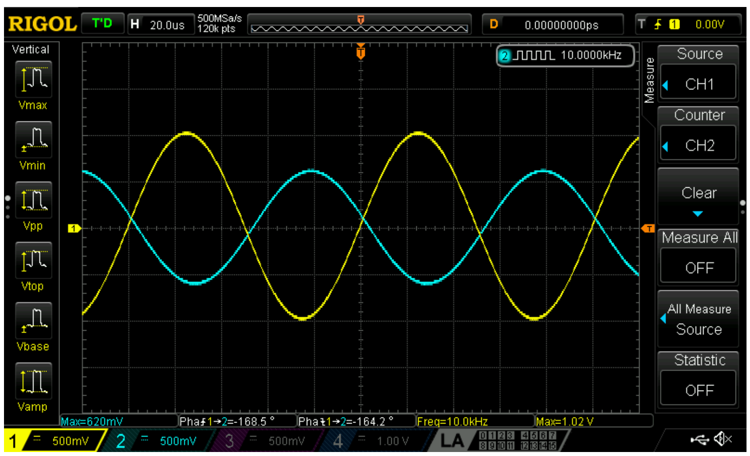


Figure 32: Step B, 10kHz



Step D Oscilloscope Results:

Figure 33: Step D, 2kHz

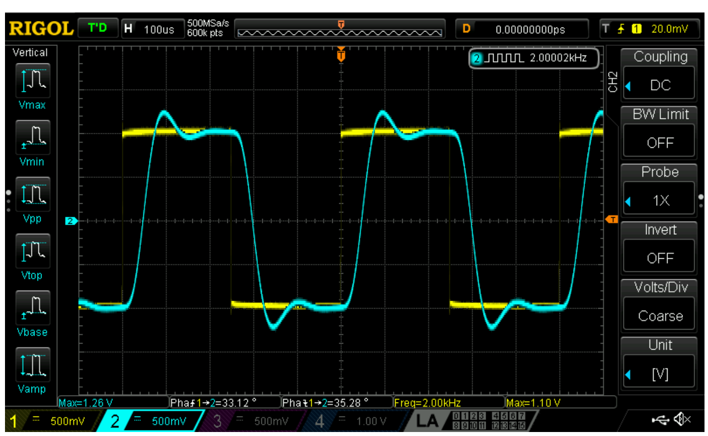


Figure 34: Step D, 3kHz

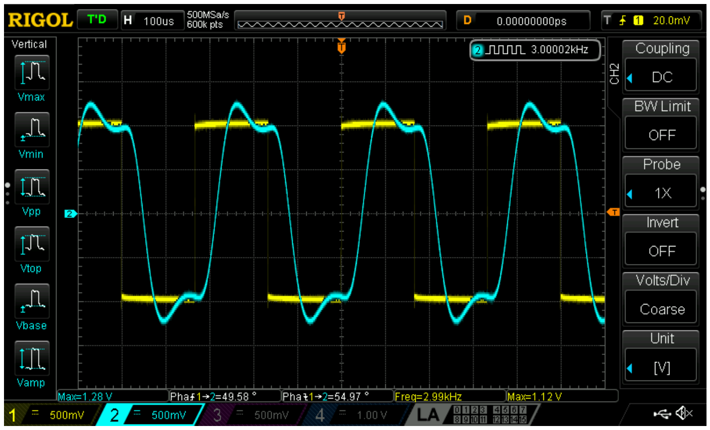


Figure 35: Step D, 4kHz

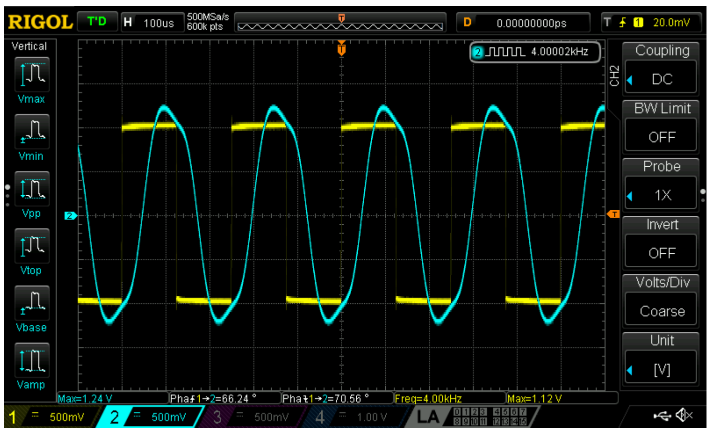


Figure 36: Step D, 5kHz

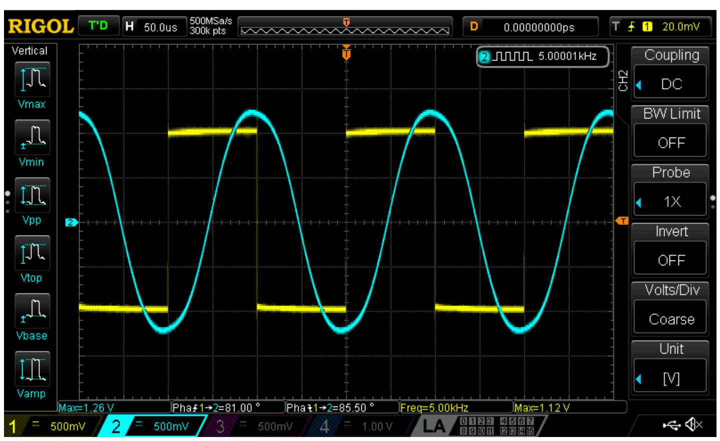


Figure 37: Step D, 10kHz